count of this enormously complex doctrine, but a few simple points will suffice for the task at hand.[13] Aristotle declares "it is what is always and everywhere that we call universal" (*Posterior Analytics* 1.31; 87b33); and in *De Interpretatione* he remarks that "of actual things some are universal, others particular (I call universal that which is by its nature predicated of a number of things, and particular that which is not; man, for instance, is an universal, Callias a particular)" (*De Interpretatione* 7; 17a38–17b). Although he did not take universals to be separately existing substances, Aristotle held that the universal is present in the particular and can be grasped by the intellect. To understand that the interior angles of a triangle sum to two right angles, the intellect must grasp the universal "triangle" and reason about it; and, in general, to understand that the individual *a* is *F*, one grasps the universal *F*-ness, and sees that it pertains to the individual *a*. Exactly how the intellect goes about grasping these abstract essences or universals—and how these relate to the mundane realm of material things—is a complicated story that can barely be sketched here. Aristotle held that sense perception involved the transmission of "sensible species" from the perceived object to the mind of the perceiver; by analogy, the intellect's grasp of universals involves its reception of "intelligible species" from its proper objects. Like many other proponents of the "mechanical philosophy" Hobbes thought that the doctrine of sensible species was nonsense to be supplanted by a mechanistic account of sensible qualities in terms of the motion and impact of material particles. His scruples extended further, however, and included the abolition of the doctrine of intelligible species in favor of his materialistic theory of the mind.[14]

13. See Largeault 1971 for a study of nominalism and its history, particularly pt. 3, sec. 2, on "the baroque nominalism of Hobbes."

14. Hobbes's attitude is well summarized in *Leviathan*, where, after summarizing his own theory of sensation, he writes:

> But the Philosophy-schooles, through all the Universities of Christendome, grounded upon certain Texts of *Aristotle*, teach another doctrine; and say, For the cause of *Vision*, that the thing seen sendeth forth on every side a *visible species* (in English) a *visible shew, apparition*, or *aspect*, or *a being seen*; the receiving whereof into the Eye is *Seeing*; and for the cause of *Hearing*, that the thing heard, sendeth forth an *Audible species*, that is, an *Audible aspect*, or *Audible being seen*; which entering at the Eare, maketh *Hearing*. Nay for the cause of *Understanding* also, they say the thing Understood sendeth forth *intelligible species*, that is, an *intelligible being seen*; which comming into the Understanding, makes us Understand. I say not this, as disapproving the use of Universities: but because

The nominalistic criticism of the theory of universals typically focuses on its bloated ontology and its attribution of seemingly odd mental powers to quite ordinary human minds. Hobbes is hardly unique in complaining that a world populated by the universal humanity, over and above individual humans, does seem rather crowded; nor is he the only philosopher to have wondered whether a proper theory of human understanding requires the postulation of such mental faculties as intellection or odd objects like intelligible species and the other accoutrements of the traditional theory of universals. In place of such an overcrowded ontology, Hobbes proposes an account of human language and demonstration that makes no commitment to the existence of anything beyond material bodies and their motions.

The universal names in Hobbesian demonstrations are not names of universal things—conceived either as separately existing archetypes to which individuals conform (which we can call real universals) or as abstract concepts in the mind of the demonstrator (which we can term conceptual universals). Hobbes makes clear his rejection of the theory of real universals in the *Elements of Law,* writing that "[t]he universality of *one name* to many things, hath been the cause that men think the *things* themselves are universal; and so seriously contend, that besides Peter and John, and all the rest of the men that are, have been, or shall be in the world, there is yet somewhat else that we call *man,* viz. *man in general,* deceiving themselves, by taking the universal, or general appellation, for the thing it signifieth" (*EL* 1.5.6; *EW* 4:22). His hostility toward a conceptualist treatment of universals is equally clear in *De Corpore* when he lists among the "causes of error" the belief that "the idea of anything is universal; as if there were in the mind a certain image of a man that is not that of some one man, but of man simply, which is impossible; for every idea is both one and of one thing; but they are deceived in that they put the name of the thing for the idea of it" (*DCo* 1.5.8; *OL* 1:53–54). This results in a theory that takes the universal to be a word given general significance by convention, that is to say by its being taken as a representative of a class of objects that resemble one another in some important respect—"One Universall name is imposed on many things, for their similitude in some quality, or other accident: And whereas a Proper Name bringeth

I am to speak hereafter of their office in a Common-wealth, I must let you see on all occasions by the way, what things would be ammended in them; amongst which the frequency of insignificant Speech is one. (*L* 1.1, 4; *EW* 3:3)

difficulty, and his defense of his view is confined largely to assertions to the effect that there is no need of a faculty of pure intellection or understanding to account for the use of universal names. At best, Hobbes defends his nominalism by claiming that the only alternative is the despised Scholastic language of intelligible species, separated essences, and formal quiddities. Notwithstanding Hobbes's failure to mount a sophisticated argument in defense of his nominalism, it is still worthwhile to investigate contemporary criticisms of his doctrine in order to clarify the philosophical and methodological stakes at issue.

We can begin by contrasting Hobbes's account of reasoning and demonstration with a more traditional theory that takes reason or intellection to be a mental faculty distinct from sensation or imagination. On such a view, sensation is a physical (one might even say physiological) process tied to the body and arising from the stimulation of sense organs; memory is the storage of previous sensory images; and imagination is nothing more than the recollection and recombination of items from memory or sensation. Neither faculty, however, is capable of "pure intellection"—which is a fundamentally nonsensory mode of awareness. Aquinas, in his adaptation of the Aristotelian theory of intellection, held that the senses provide the material upon which the intellect operates; intellectual understanding arises when the "active intellect" abstracts the intelligible species from sensible species. He reasons that

> [b]ecause Aristotle did not allow that forms of natural things subsist apart from matter, and since forms existing in matter are not actually intelligible, it follows that the natures or forms of the sensible things we understand are not actually intelligible. But nothing is reduced from potentiality to actuality except by something in actuality; just as the senses are made actual by what is actually sensible. We must therefore assign on the part of the intellect some power to make things actually intelligible, by the abstraction of the species from material conditions. And such is the necessity for positing an agent intellect. (Aquinas 1964, 11:154)

One consequence of this view is that "on the part of the phantasms, intellectual operations are caused by the senses. But because the phantasms cannot of themselves change the passive intellect, but require to be made actually intelligible by the active intellect, it cannot be said that sensible knowledge is the total and perfect cause of intellectual knowledge, but rather it is in a way the material cause" (Aquinas 1964, 12:36).

We need not be overly concerned with the details of this theory, but it is important to recognize the extent to which it separates the intellect from the senses. Intellectual understanding, although it may make use of sensory ideas as inputs, has as its proper object universals or essences that exceed the grasp of the senses. Barrow phrases his version of this theory by putting intellectual powers in analogy with the senses when he asks: "What if I should assert that the human mind (when rightly constituted and not out of balance, as it is in extreme fools and demented people) has the power of discerning universal propositions by its native faculty, in the same manner as sense discerns particular ones? . . . Such universal propositions the mind directly contemplates and finds to be true by its native power, even without any previous notion and having applied no reasoning; which means of attaining truth is called by the peculiar name intellection [νόησις], and this faculty the intellect [νοῦς]" (*LM* 5, 81). Ward makes a similar case for the immateriality of the intellect when he argues that consideration of such ideas as those of God or the soul "are sufficient to shew the difference betwixt the intellectuall apprehension of things, and the imagination which accompanies our superficiall thoughts, our slight and cursory taking of them to our mindes, and to illustrate that, however in our fancies we may have corporeall representations attending upon these spiritual beings, yet the Ideas whereby the understanding apprehends those simple essences, are incorporeall, and consequently the understanding part of man is incorporeall" (Ward 1652, 57–58).

5.1.2.1 DESCARTES VS. HOBBES ON REASON AND THE INTELLECT The most famous seventeenth-century proponent of this sort of theory is Descartes, who notoriously contrasts the powers of "pure understanding" with those of the imagination in the sixth of his *Meditations*. He there claims that the intellect can frame the idea of a thousand-sided figure—the chiliagon—and even demonstrate necessary properties of it, although the idea of such a figure is too complex to be framed distinctly in imagination (*AT* 7:72). In constructing its purely intellectual idea of the chiliagon the intellect grasps the essence of the figure, and does so without the mediation of sensory ideas. This is not to say that, for Descartes, the intellect lacks all connection to the senses or imagination. In a letter to Princess Elisabeth in June of 1643, Descartes explains that the study of mathematics "exercises mainly the imagination in the consideration of figures and motions" (*AT* 3:691), and the ideas furnished by the imagination provide the intellect with the material from which it can abstract essences. Nevertheless, Des-

Descartes wholeheartedly opposed nominalism of this sort, and he seems to have taken particular umbrage at the suggestion that reasoning can tell us nothing about things themselves. Cartesian pure thought or understanding grasps the true nature of its objects and does so without relying upon linguistic devices or the manipulation of names. Descartes responded to the objection by insisting that Hobbes had wholly misconstrued the nature of reasoning—it "is not a linking of names, but of the things that are signified by the names." Further, he charged that his English opponent "refutes his own position when he talks of the arbitrary conventions that we have laid down concerning the meaning of the words. For if he admits that the words signify something, why will he not allow that our reasoning deals with this something that is signified, rather than merely with the words?" His frustration with the Hobbesian enterprise ultimately led Descartes to remark that "when [Hobbes] concludes that the mind is motion he could just as well conclude that the earth is the sky, or whatever else he likes" (*OL* 5:258–59; *AT* 2:179).

At its most fundamental level this conflict between Hobbes and Descartes concerns the delicate question of what we think about when we understand a demonstration. For Hobbes, the objects of thought are always concrete particular phantasms (including names, which themselves are "sensible marks"). These phantasms represent the world of external bodies to us; thus, my present greenish visual sensation represents the tree outside my window. Our words do not signify external bodies directly, but rather our thoughts, since "it is manifest that they are not signs of things themselves; for in what sense can the sound of the word 'stone' be a sign of a stone, except that he who hears the word concludes that the speaker thinks of a stone?" (*DCo* 1.2.5; *OL* 1:15). Although its immediate signification is not an external body itself, a name does designate a body indirectly, by signifying a phantasm of it, which in turn represents the body itself. Hobbes therefore often speaks of names as designating the things themselves and omitting the phantasms that forge the link between words and the world. As we have already seen, Hobbes holds that reasoning involves the manipulation of general names (which represent classes of things that resemble one another in some quality). Linking together such names by use of the verb *is* we form syllogisms, whose conclusions yield additional information about the consequences of the names we have imposed. In no case, however, does our reasoning deal either directly with the external objects or with any universal ideas or nonphantasmic intellectual notions.

Descartes holds that in demonstration the intellect deals with its proper object, i.e., concepts or essences that are eternal, immutable, universal, and independent of sense. Except in the case of God, the essence and existence of a thing are distinct, which is to say that from the bare essence of the thing no conclusion follows about whether such a thing exists. Demonstrations therefore need not tell us anything about what the world contains, but they do make manifest the relationship between the essences of things that can exist. Demonstration consequently allows us to gain knowledge about the way things must be structured, if they exist at all. The essence of the semicircle, for example, entails that every triangle inscribed in it is a right triangle, and this result is independent of the structure or contents of the physical world, which may not contain any true circles or triangles. Because "pure thought" or intellect is independent of matter, its objects are also immaterial, and the Cartesian demonstrator, insofar as he attends to his intellectual ideas, is radically separated from the world of bodies. The connection between the demonstrator and the material world is forged by the imagination, which furnishes determinate ideas to the otherwise indeterminate essences of the intellect.[17] Thus, the Cartesian demonstrator may employ visible diagrams in his geometric investigation, and these may assist him in grasping the immaterial essences that are the true objects of his concern. Nevertheless, the business of drawing conclusions and comprehending essences is proper to the intellect alone. It is for this reason that Descartes can hold that in reasoning we deal with the things themselves, not merely names. These "things themselves" turn out to be essences immediately graspable by the intellect and necessarily governing the structure of the world.

Descartes's doctrine of essences found no favor with Hobbes. When Descartes declares that the fact that he can demonstrate properties of the triangle shows that "there is a determinate nature, essence, or form of the triangle, which is immutable, eternal, and not made by me or dependent on my mind," and infers that the nature of the triangle would remain even if there were no triangles outside of his thought,

17. Gaukroger 1992, 110, describes this process in the following terms: "It is [in the connection between intellect and the material world] that the necessity for the imagination arises, because the intellect by itself has no relation at all to the world. Entities conceived in the intellect are indeterminate. The imagination is required to render them determinate. When we speak of numbers, for example, the imagination must be employed to represent to ourselves something that can be measured by a multitude of objects. The intellect understands 'fiveness' as something separate from five objects (or line segments, or points, or whatever), and hence the imagination is required if this fiveness is to correspond to something in the world."

Hobbes replies: "If the triangle exists nowhere, I do not understand how it has a nature." In Hobbes's scheme, talk of eternal essences can at best be understood as applying to names. The name *triangle* can be applied to something we have seen, and "if in our thought we have once conceived that the sum of all the angles of a triangle taken together equals that of two right angles, and we give the triangle a second label 'having three angles equal to two right angles,' then even if no angles existed in the world, the name would remain." This allows the truth of the proposition "every triangle has angles summing to two right angles" to be distinguished from the existence of any particular triangle, so that "the truth of the proposition will be eternal."[18] But there can be no immutable essence of a triangle, "if it should happen that every single triangle ceased to exist." Hobbes concludes that any notion of essence "insofar as it is distinct from existence, is nothing more than joining together of names by means of the verb *is;* and so essence without existence is our mental fiction." Needless to say, Descartes was unimpressed with this theory, and in his reply to the objection he simply states that "The distinction between essence and existence is known to everyone; and this talk about eternal names, rather than concepts or ideas of eternal truths, has already been fully refuted" (*OL* 5:271–72; *AT* 7:193).

5.1.2.2 WARD'S CRITIQUE AND THE LIMITS OF MATERIALISM I take it to be unproblematic that there is a significant connection between Hobbes's nominalism and his materialism.[19] By rejecting any theory of pure intellect and insisting upon the adequacy of the imagination to account for our understanding of universal terms, Hobbes rules out the possibility of "higher" mental faculties of the sort that had traditionally been taken as immaterial. Nevertheless, nominalism of this sort does not lead to materialism all on its own. It is only when conjoined with a materialistic account of sensory ideas (and hence of the imagination) that Hobbes's nominalism becomes indissolubly linked with his materialistic theory of the mind. To see this point more clearly it is helpful to recall that George Berkeley champi-

18. Hobbes cannot be too serious about the locution "eternal truth" here, since eternal truth presupposes the existence of eternal names and there is no guarantee that the names forming the proposition will be eternal. Presumably he would hold that when the human race dies out and the world's geometry books crumble to dust, the proposition will vanish with the words in which it was expressed.

19. On the connection between Hobbes's nominalism and materialism, see Zarka 1985 and Zarka 1987, chap. 4.

there is no need of any faculty beyond the imagination. This principle, he says, would be sufficiently established if it could be shown that "the concept corresponding to a universal name is the phantasm of a singular thing (although the phantasm of the singular thing does require the imagination)" (*Exercitatio* 30).

Ward takes up the question of what sort of argument might prove that the grasp of a universal name requires only the phantasm of a singular thing. He reports that "either, says Hobbes (for this is the import of his argumentation), the concept corresponding to a universal name is some singular phantasm, or by a universal name is signified that there is something universal in nature that is designated by this name. For example either the concept corresponding to this word 'man' is a phantasm of Peter, or John, etc. or there is some universal man to whom this word corresponds" (*Exercitatio* 30). Since the second alternative is plainly ridiculous, the conclusion follows.

Ward attacks the soundness of the above argument and mounts a challenge to the entire Hobbesian conception of mental faculties. His strategy is to accuse Hobbes of invoking a false dilemma in assuming that the meaning of a universal term must be either a singular phantasm or a universal thing. Instead of these two options, he introduces a third, namely the concept of a "common reason" *[ratio communis]* shared by the particulars that fall under a universal name.

> Who does not see how lax, how fluid, and how completely unsound this type of argument is, which this upstart offers for sale under the name of demonstration? The consequence is this: either the concept corresponding to this word (man) is the concept of Peter, etc. or it is the concept of some common reason, in which Peter, John, and the rest are included, that is, a common and explicable concept of Peter, etc. And this concept is not the phantasm of a singular thing, nor is it the concept of some universal man existing in nature, but the idea of a certain common reason underlying the same species of all the singulars, the formation of which idea can be in no way taken from the imagination, which deals with singulars. (*Exercitatio* 30)

The battle against Hobbes clearly requires more than the statement of the theory of a "common reason," however. Ward promises that "in

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and considered language *[aperto capite & verbis conceptis]* in *Leviathan:* that the soul dissolves at death, and that those who hold the contrary opinion are not to be tolerated in the commonwealth, on account of the disorder, sedition, and civil wars that arise from this empty opinion of immortality" (*Exercitatio* 29).

order to explain my own opinion and bring Hobbes's tricks to light, I will set forth the method of common human reason in the framing of universals" (*Exercitatio* 30–31).

Indeed, he argues that the inadequacy of nominalism follows from the careful consideration of a case that Hobbes himself had advanced in support of his own theory—that of a person who proves a geometric theorem by attending to the properties of a single triangle. Hobbes had argued that by the imposition of names (such as *triangle* and *equal*) the consideration of a single case could lead to the general result that all triangles have interior angles that sum to two right angles, since "he that hath the use of words, when he observes, that such equality was consequent, not to the length of the side, nor to any other particular thing in his triangle . . . will boldly conclude Universally, that such equality of angles is in all triangles whatsoever" (*L* 1.4, 14; *EW* 3:22). Ward argues that this is an inadequate description of the reasoning involved in a demonstration. He insists that any such particular triangle must be completely specific—it will have a determinate color and position, and its sides will be of specific lengths. The result concerning the equality of the angles can be discerned, but he thinks that there are two ways of doing this: "either by experience (so much as the equality of things can be known by sense), for example by aid of a pair of compasses measuring the angles of the arcs of circles; or it can be found by a deep and attentive *[alta et defixa]* cogitation of the soul" (*Exercitatio* 31).

Ward contends that, in the first case, the reasoning is essentially tied to singular phantasms derived from experience, and there is no chance that the result can be extended to cover other cases. The experience of a particular triangle includes such features as color and situation and is therefore too specific to underwrite a general result. At best such an experience can be taken to show that this individual triangle has angles equal to two right angles, but it says nothing about other cases. To obtain a general result the demonstrator must disregard irrelevant features of this specific triangle and see that the proof depends only upon the "common reason" of all triangles. But such generalization involves higher mental powers than simply imagination. Ward holds that it is manifest that the reasoning involved here "exceeds the power of the imagination, for this does not create ideas, but only receives them from objects, nor can it move beyond the sphere of objects," because anything perceived or imagined must include determinate color, shape, size, etc. He reasons that

mathematics (especially geometry) and such natural-philosophical inquiry as physics. The geometer begins his demonstration with definitions that express the true causes of the objects of his inquiry.[21] In contrast, the causes of natural phenomena are generally hidden from us and mathematics consequently has a degree of clarity and certainty exceeding that obtainable in natural philosophy. Because Hobbes holds that our knowledge of nature is confined to "phantasms" or "fancies" in the mind, which in turn are caused by the motion of external bodies, the task of natural philosophy is thus to find a mechanistic explanation of such phantasms. "Your desire," he says to the prospective natural philosopher "is to know the causes of the effects or phenomena of nature; and you confess they are fancies, and consequently, that they are in yourself; so that the causes you seek for only are without you, and now you would know how those external bodies work upon you to produce those phenomena" (*EW* 7:82)

It is axiomatic for Hobbes that "nature does all things by the conflict of bodies pressing each other mutually with their motions" (*DP* epistle; *OL* 4:238). Nevertheless, there are many possible accounts that may satisfy this general requirement of mechanism since "there is no effect in nature which the Author of nature cannot bring to pass by more ways than one" (*EW* 7:88). The result is that natural science must proceed from conjectures about the possible generation of natural phenomena, so there must be an ineradicably conjectural or hypothetical aspect to natural science. The essentially hypothetical nature of natural science leads Hobbes to conclude that—contrary to the tradition—there can be no demonstration τοῦ διότι in physics. Furthermore, since he denies that τοῦ ὅτι "demonstrations" are properly demonstrations at all, Hobbes concludes that there is no hope for a truly demonstrative science of nature. He brings this point out in the *Examinatio*, when he insists that "reasoning that, beginning from true principles, correctly infers a conclusion is properly called demonstration. Nor do I think that Aristotle called reasoning in which there is a paralogism 'demonstration,' not even a τοῦ ὅτι demonstration. And this is how he must have understood reasoning that begins not with definitions but with suppositions (such as physicists use), which are generally uncertain" (*Examinatio* 1; *OL* 4:38). It is in precisely this context that Hobbes made his famous declaration that

21. This should be taken to apply most specifically to Hobbes's own program for properly reformed geometry. Obviously, he does not think that Euclid or those who follow him begin from principles expressing true causes.

> [o]f Arts, some are demonstrable, others indemonstrable; and demonstrable are those the construction of the Subject whereof is in the power of the Artist himself; who in his demonstration does no more but deduce the Consequences of his own operation. The reason whereof is this, that the Science of every Subject is derived from a praecognition of the Causes, Generation, and Construction of the same; and consequently where the Causes are known, there is place for Demonstration; but not where the Causes are to seek for. Geometry therefore is demonstrable; for the Lines and Figures from which we reason are drawn and described by ourselves; and Civill Philosophy is demonstrable because, we make the Commonwealth our selves. But because of Naturall Bodies we know not the Construction, but seek it from the Effects, there lyes no demonstration of what the Causes be we seek for, but onely of what they may be. (*SL* epistle; *EW* 7:183–84)

A similar passage in *De Homine* even holds out the hope that the science of geometry can be made complete, in the sense that there could be no unanswerable question regarding figures. Hobbes declares that "many theorems concerning quantity are demonstrable, the science of which is called geometry. Since the causes of the properties that individual figures have are in them because we ourselves draw the lines, and since the generation of the figures depends on our will, to know the properties belonging to any figure whatsoever nothing more is required than that we consider all that follows from the construction that we ourselves make in the figure to be described" (*DH* 2.10.5; *OL* 2:93). It is Hobbes's faith in such definitions that led him to underestimate the difficulty of the classical problems such as the quadrature of the circle and to imagine that their solution would be available to one who had grasped the proper definitions of figures in terms of the motions by which they are produced.

Hobbes's doctrines assimilate mathematics (and demonstrative knowledge generally) into the domain of "maker's knowledge," by grounding its certainty and universality in our construction of the objects known. Some commentators have seen this aspect of Hobbes's philosophy as an inheritance from Francis Bacon, at least to the extent that Bacon saw scientific knowledge as maker's knowledge.[22] Whether

22. On Bacon and the tradition of "maker's knowledge," see Perez-Ramos 1988, esp. 186–93, on the methodological connection between Bacon and Hobbes. Barnouw (1980) argues for a fairly strong Bacon-Hobbes connection, as does Child (1953). Bernhardt (1989, 10) remarks that the relationship between the two is an "obscure point"

such claims of influence ultimately hold up is not a matter I will investigate here, but it is worth noting that Bacon does not hold that mathematical objects are constructions founded in the nature of body, and this part of the Hobbesian program cannot claim a strictly Baconian pedigree.

There is certainly nothing novel or remarkable in Hobbes's claim that mathematics is more certain than natural science. The traditional distinction between pure and applied mathematics, for example, places pure mathematics on a lofty plane of metaphysical certainty, rendering its truths unencumbered by dependence upon the features of the material world. Applied mathematics, such as that in optics or astronomy, introduces physical hypotheses that permit material objects to be treated mathematically, and such hypotheses depend for their truth upon contingent facts about the material world. Thus, the science of optics can be developed "mathematically" by the hypotheses that light is propagated in straight lines and its rays behave in accordance with Euclidean geometry, although it is possible that such hypotheses might fail to be true. Even Descartes, who was not one to regard his physics as so much fallible conjecture, drew a distinction of this sort in a 1638 letter to Mersenne:

> You ask whether I take what I have written about refraction to be a demonstration. I think so, at least insofar as one can be given in these matters without having previously demonstrated the principles of physics by metaphysics (which I hope to do some day, but which has not yet been done) and insofar as any other solution has been demonstrated to a problem of mechanics, optics, astronomy, or anything else that is not pure geometry or arithmetic. But to require me to give geometrical demonstrations in matters that depend on physics is to want me to do the impossible. And if you will not call anything demonstrations except the proofs of geometers, then you must say that Archimedes never demonstrated anything in mechanics, or Vitellio in optics, or Ptolemy in astronomy, which of course nobody says. In such matters it is enough if the authors have supposed things not obviously contrary to experience and if their discussion is coherent

in Hobbes's intellectual biography. Tönnies (1925, preface) launches a vigorous polemic against any reading of Hobbes as a successor to Bacon, a view shared by Brandt (1928). Schumann 1984 contains a nuanced overview of the difficult question of Hobbes's relationship to Bacon. My own view is that there is little connection between Bacon and Hobbes on the specifically mathematical issues of concern to us here.

and without paralogism, even though their assumptions may not be strictly true. (*AT* 2:141–42)

Hobbes's contrast between conjectural physics and certain mathematics might seem to comport poorly with his placement of such plainly physical concepts as body, space, time, and motion at the foundation of his mathematics. We have already seen that Hobbes makes these concepts central to his philosophy of mathematics, and one of Wallis's charges against Hobbes is that he makes mathematics overly dependent upon (uncertain) physical principles. How, one might ask, can Hobbes take physical notions as basic to mathematics, and yet claim demonstrative certainty for mathematics while denying it to physics?

The resolution of this apparent contradiction lies in the recognition that Hobbes regards these concepts as basic, not only to mathematics and physics, but to any body of knowledge whatsoever. They are, in fact, the first principles of metaphysics.[23] As such they are known more clearly than other concepts and serve as the basis upon which any science (demonstrative or hypothetical) must be built. The actual derivation of physical phenomena from the first principles is not an option for us. We did not create the world and its phenomena, and in natural philosophy "the most that can be atteyned vnto is to haue such opinions, as no certayne experience can confute, and from w[ch] can be deduced by lawfull argumentation, no absurdity" (Hobbes to William Cavendish, 29 July/8 August 1636; *CTH* 1:33). Mathematical objects, however, are our constructions, and we can have clear, certain, and demonstrative knowledge of their properties because we know how they are generated. In fact, that part of natural philosophy that concerns the general science of motion is capable of demonstration. In *De Homine,* Hobbes distinguishes the a posteriori part of physics (which is purely hypothetical) from the a priori part, which is purely geometrical: "Since one cannot proceed in reasoning about natural things that are brought about by motion from the effects to the causes without a knowledge of those things that follow from every kind of motion; and since one cannot proceed to the consequences of motions without a knowledge of quantity, which is geometry; nothing can be demon-

23. It is in this context that Hobbes declares "words understood are but the seed, and no part of the harvest of Philosophy." He then contrasts his fundamental definitions with those of Aristotle, concluding that "this is the Method I have used, defining Place, Magnitude, and the other most generall Appellations in that part [of *De Corpore*] which I intitle *Philosophia prima*" (*SL* 2; *EW* 7:226).

strated by physics without something also being demonstrated a priori" (*DH* 2.10.5; *OL* 2:93). Nevertheless, there is no science of "sensible appearances" because the specific causal mechanisms that produce such appearances remain hidden from our view.[24]

5.2 ANALYSIS, SYNTHESIS, AND MATHEMATICAL METHOD

Hobbes's theory of demonstration provides the background for his version of the distinction between analytic and synthetic methods—a topic of great importance in any study of his mathematical writings. It goes without saying that this distinction did not originate with Hobbes, and indeed some kind of contrast between analysis and synthesis was a commonplace well before the seventeenth century, especially in the philosophy of mathematics.[25] The volume of primary and secondary literature on this topic makes it impossible to treat it in its full complexity, and I will be content to investigate that part of it that bears directly on Hobbes. In point of fact the confusion generated by the many different pronouncements on the nature of analysis and synthesis is so great that one could be forgiven the suspicion that every author who held forth on the subject had his own way of distinguishing the two methods.[26] I begin this section with a quick summary of the

24. Zarka (1996, 73) makes this point as follows:

> One could say that the concepts of philosophy are perfectly adapted to grounding the physical sciences, as long as one distinguishes between the science of motion and the science of the sensible world. The science of motion considers in the abstract the effects of one body on another, that is, the laws of impact, and more generally, the laws of the transmission of motion. On the other hand, the science of the sensible world, which Hobbes thinks is physics properly so-called, concerns what appears to the senses and the causes of these appearances. The former science is elaborated a priori from the concepts of first philosophy, while the latter, being concerned with sensible appearance, depends on hypotheses arrived at a posteriori.

25. See Hintikka and Remes 1974 for a study of the method of analysis and its history. More specifically on Hobbes's conception of analysis and synthesis, see Talaska 1988. Hanson 1990 is a very useful study of the role of demonstration in Hobbes's methodology, especially as concerns the issue of analysis and synthesis.

26. Rashed sums up the confusion nicely with the following observation: "Among the problems on the border of philosophy and mathematics, that of analysis and synthesis has occupied a central place for two millennia. Rare indeed are the problems in the philosophy of mathematics which have survived for so long and have given rise to so many writings. Present in shadow in the writings of Aristotle, it is there in person in the works of commentators, of philosophers, and of logicians until the beginning of the last century. It is easy to imagine the diversity of senses and the multiplicity of formulations

classical understanding of analysis and synthesis, then move on to outline how the doctrine was interpreted in the seventeenth century. An exposition of Hobbes's own version of the methods of analysis and synthesis then follows, after which I consider the extent (if any) to which Hobbes's doctrines are an inheritance from the sixteenth-century Paduan school, and specifically the writings of Jacopo Zabarella.

5.2.1 Classical Sources of the Analysis-Synthesis Distinction

The roots of the methods of analysis and synthesis lie in the classical discussions of the philosophy of mathematics, and any discussion of the topic inevitably leads back to a rather small number of classical texts. In fact, it was customary in Hobbes's day to claim a classical pedigree for the distinction. François Viéte's 1591 *In artem analyticam isagoge* is typical. It opens with the declaration that

> there is a certain way of searching for the truth in mathematics that Plato is said first to have discovered; Theon named it analysis, and defined it as the assumption of that which is sought as if it were admitted and working through its consequences to what is admitted as true. This is opposed to synthesis, which is the assuming what is admitted and working through its consequences to arrive at and to understand that which is sought. (Viéte 1646, 1)

Whether Plato can be credited with this methodological discovery is unclear, but there is no doubt that philosophical conceptions had much to do with the framing of the contrast between analysis and synthesis.[27] Aristotle's account of deliberation in the *Nicomachean Ethics* contains an important passage that links analysis with the search for means to a sought end:

> We deliberate not about ends but about what contributes to ends. For a doctor does not deliberate whether he shall heal, nor an orator whether he shall convince, nor a statesman whether he shall produce law and order, nor does any one else deliberate

of this question of analysis and synthesis, which then designated a domain vast enough to encompass at once an *ars demonstrandi* and an *ars inveniendi*" (1991b, 131–32).

27. Proclus is presumably the source for attributing the method of analysis to Plato, when he remarks that "there are certain methods [for the discovery of lemmas] that have been handed down, the best being the method of analysis, which traces the desired result back to an acknowledged principle. Plato, it is said, taught this method to Leodamas, who also is reported to have made many discoveries in geometry by means of it" (Proclus 1970, 165–66).

> about his end. Having set the end, they consider how and by what means it is to be attained; and if it seems to be produced by several means they consider by which it is most easily and best produced, while if it is achieved by one only they consider how it will be achieved by this and by what means this will be achieved, till they come to the first cause, which in the order of discovery is last. For the person who deliberates seems to inquire and analyze in the way described as though he were analyzing a geometrical construction (not all inquiry appears to be deliberation—for instance mathematical inquiries—but all deliberation is inquiry), and what is last in the order of analysis seems to be first in the order of becoming. And if we come on an impossibility, we give up the search, e.g., if we need money and this cannot be got; but if a thing appears possible we try to do it. (*Nicomachean Ethics* 3.3; 1112b11–26)

This version of the distinction takes analysis to be a kind of "working backwards" from what is sought, with the intention of finding principles from which the desired end can be produced. Synthesis would then appear to be a process of generation, by which the desired result is constructed from first principles (or at least known principles sufficient to generate the thing sought). Proclus, in a rather offhand remark in the first part of his commentary on the *Elements*, connects analysis and synthesis to a broadly Aristotelian theory of demonstration when he says that "[c]ertainly beauty and order are common to all branches of mathematics, as are the method of proceeding from things better known to things we seek to know, and the reverse path from the latter to the former, the methods called analysis and synthesis" (Proclus 1970, 6–7). The epistemic distinction between what is more or less known recalls Aristotle's famous dictum in the *Physics* that the "natural path" of inquiry into principles "is to start from the things which are more knowable and clear to us and proceed towards those which are clearer and more knowable by nature; for the same things are not knowable relative to us and knowable without qualification" (*Physics* 1.1; 184a16–19). The things best known to us turn out to be "inarticulate wholes" that can be analyzed into their elements and principles, thereby leading us back to those things that are better known to nature.

The most complete account of the doctrine of analysis and synthesis in classical literature comes from book 7 of Pappus's *Mathematical Collection*. This book contains a summary of the so-called "treasury

the only one; most of the ancients hid their analytics (for it is beyond doubt that they had one) from their posterity" (Wallis 1658, 43).

Wallis also followed the lead of Viète in seeing the development of algebra as a significant point of superiority for the new methods over the old. In fact, he was prepared to identify algebra with the method of analysis—seeing in it a general technique that can yield the solution to problems of any kind. He frequently refers to "the universal algebra or analytics," or to "the analytic or algebraic method" in the solution of problems (*MU* 11; *OM* 1:53, 59). In the "Inaugural Oration" delivered upon his assumption of the Savilian chair, for example, Wallis gives a brief sketch of the history of mathematics and says of the current era that "beyond the many theorems lately discovered, much is facilitated by this method of discovery. That is to say, algebra, or the analytic practices beyond what were known to the ancients, are now becoming known" (*OM* 1:8). In his *Treatise of Algebra* Wallis announces that his work

> contains an Account of the Original, Progress, and Advancement of (what we now call) *Algebra*, from time to time; shewing its true Antiquity (as far as I have been able to trace it;) and by what Steps it hath attained to the Height at which now it is.
>
> That it was in use among the *Grecians*, we need not doubt; but studiously concealed (by them) as a great Secret.
>
> Examples we have of it in *Euclid*, at least in *Theo*, upon him; who ascribes the invention of it (amongst them) to *Plato*.
>
> Other Examples we have of it in *Pappus*, and the effects of it in *Archimedes*, *Apollonius*, and others, though obscurely covered and disguised. (*Treatise of Algebra* preface, sig. a2)

Hobbes was particularly annoyed by the identification of analytic methods and algebra, as we will see in section 5. 3. But in order to make sense of his complaints against "the modern analytics" we must proceed to an account of his version of the distinction between analysis and synthesis.

5.2.3 Hobbes on Analytic and Synthetic Methods

Hobbes phrases his version of the analytic-synthetic distinction in terms of causes and effects. Thus, in the Hobbesian scheme, the difference between analysis and synthesis lies in the comparison between the order of reasoning and the order of cause and effect: to reason analytically is to proceed from effects to causes, while the synthetic mode of reasoning follows the natural causal order and moves from

causes to effects. As Hobbes himself expresses the contrast in *De Corpore:*

> analysis is ratiocination from the supposed construction or production of a thing to the efficient cause or many coefficient causes of what is produced or constructed. And so synthesis is ratiocination from the first causes of a construction, continued through middle causes to the thing itself that is produced. (*DCo* 3.20.6; *OL* 1:254)

This talk about causes and effects can be interpreted in terms of the premises and conclusions of a demonstration, since proper demonstrations must have premises which express causes. In his discussion of the "analysis of the geometricians" in *De Corpore,* Hobbes clarifies this connection between causes and definitions. He first notes that analytic reasoning "tends ultimately to some equation" so that "there is no end of resolving until at last the very causes of equality and inequality are arrived at, or theorems previously demonstrated from those causes, and enough of them to demonstrate what was sought" (*DCo* 3.20.6; *OL* 1:253). The search for causes must ultimately lead back to definitions:

> And seeing also, that the end of the analytics is either the construction of a possible problem or the detection of the impossibility of a construction; if the problem is possible, the analyst must not stop, until he comes across things in which are contained the efficient cause of what he is to construct. But he must of necessity stop when he comes to first propositions, and these are definitions. In these definitions therefore there must be contained the efficient cause of the construction; I say of the construction, not of the demonstrated conclusion. For the cause of the conclusion is contained in the premised propositions, that is, the truth of the proposition proved is in the propositions that prove it. But the cause of the construction is in these things, and consists in motion or the concourse of motions. Therefore, the propositions in which analysis terminates are definitions, but of such kind as signify the manner in which the thing itself is constructed or generated. (*DCo* 3.20.6; *OL* 1:253)

In order to convert an analysis to a synthesis from such definitions, it is necessary that the steps in the analysis be reversible, in the sense that there is a logical entailment from each result obtained by analysis to its antecedent step. In Hobbes's terminology this requirement means

that "the terms of all the propositions should be convertible; or if they are enunciated hypothetically, it is necessary not only that the truth of the consequent follow from the truth of its antecedent, but also that the truth of the antecedent be inferred from the truth of the consequent." Failing this, "when by resolving the principles are arrived at, there is no reverse composition to what is sought" (*DCo* 3.20.6; *OL* 1:252).

Hobbes defines all philosophy (i.e., demonstrative *scientia*) as "*such knowledge of effects or appearances as we acquire by true ratiocination from the knowledge we have first of their causes or generation. And again, of such causes or generations as may be from knowing first their effects*" (*DCo* 1.1.2; *EW* 1:3). Thus, he holds that neither analytic nor synthetic methods are confined to mathematics, but that both can be employed throughout all branches of philosophy. Indeed, Hobbes claims that both modes of reasoning are necessary in any investigation of causes.[32] Mathematics has a place for both analysis and synthesis in the course of proving theorems and solving problems: one proceeds analytically by first assuming what was to be demonstrated and then investigating the conditions necessary for its demonstration. Then, provided that all of the steps in the analysis are reversible, a synthetic demonstration from first principles can be effected. Here the analytic method functions as a preface to synthesis and is intended to aid in uncovering causal principles that can then be used to generate true demonstrations. Should it happen that the analysis leads to an absurdity, then the supposition at the beginning of the analysis is shown to be false.

The differences between mathematics and natural philosophy will be reflected in the kinds of analyses that can be performed in each area. Natural science must begin with phantasms or fancies in the mind whose existence is not in question.[33] Thus, unlike the mathematical

32. I leave aside the question of whether Hobbes sees a true unity of method between natural science and political science. Sorell (1986) argues that there is a methodological disunity between natural and civil science, notwithstanding Hobbes's claims that the methodology of analysis and synthesis is universally applicable. To whatever extent Hobbes is serious about his claim that the business of philosophy is the investigation of causes, it seems reasonable that he would see analytic and synthetic procedures as integral to the philosophical enterprise. Hanson (1990) addresses the question of how far Hobbes's talk of method commits him to the idea that there is a unity of method across disciplines.

33. As Hobbes puts it, "Thus the first principles of all knowledge are the phantasms of sense and imagination, which we know by nature to exist; but to know why they are

case, an analysis in natural science does not run the risk of terminating in absurdity. Analysis in natural science will lead from observed phenomena or phantasms to their possible causes, and synthesis will proceed from causal hypotheses (namely particular motions of bodies) to derivations of their effects (i.e., the phenomena to be explained). But because the causes of natural phenomena can only be the matter for hypothesis or conjecture, the synthetic procedures of the natural scientist will fall short of the demonstrative certainty obtainable in mathematics. In other words, the synthesis available to the natural philosopher will typically be no more than a synthesis from a hypothetical cause, whereas the geometer constructs his synthesis from the true causes of the objects of his investigation.

This conception of analysis as a preface to synthesis is in keeping with the traditional characterization of analysis as the "method of invention or discovery" while synthesis is the true "method of demonstration." Hobbes explicitly endorses this representation of the distinction, holding that it is by analysis that we are led "to the circumstances conducing separately to the production of effects," while synthesis takes us "to that which they effect singly by themselves when compounded together" (*DCo* 1.6.10; *OL* 1:70). Hobbes actually characterizes the synthetic method as the proper method of teaching, as it begins with uncontestable first principles and proceeds by proper syllogisms to the required result: "Therefore the whole method of demonstration is synthetic, consisting in the order of speech that begins from primary or most universal propositions, which are self-evident, and through the continual composition of propositions into syllogisms until the truth of the conclusion sought is understood by the learner" (*DCo* 1.6.12; *OL* 1:71).

Hobbes offers his explanation of light as an example of the proper employment of analysis and synthesis in natural philosophy. First we observe that there is a "principal object" or source of light whenever light is observed; by analysis, we take such an object as causally necessary to the production of light; further analysis shows that a transparent medium and functioning sense organs are required for the phenomenon to present itself. As the analysis continues we infer that a motion in the object is the principal cause of light, and the continuation of this motion through the medium and its subsequent interaction with the

or by what causes they are produced there is need of ratiocination, which consists . . . in composition and division" (*DCo* 1.6.1; *OL* 1:59).

"vital motion" of animal spirits in the sense organs are contributing causes. The result is that "in this manner the cause of light can be made up out of motion continued from its origin to the origin of vital motion, the alteration of which vital motion by this incoming motion is light itself" (*DCo* 1.6.10; *OL* 1:69–70). The analysis must stop somewhere, and Hobbes holds that it will terminate in simple, universal things that are self-evidently known and causally sufficient for the explanation of the phenomena. In the case of natural science, the motion of bodies is that self-evident principle upon which the explanation of physical phenomena depends.

Once the analysis has been completed and the phenomenon to be explained is traced back to certain motions, the way is open for a proper synthesis from first principles. This picture of method connects with the account of demonstrations *τοῦ ὅτι* and *τοῦ διότι,* since "in knowledge of the ὅτι, or that *a thing is,* the search begins with the whole idea. On the other hand, in our knowledge of the *διότι,* or of the knowledge of causes, that is, in the sciences, the parts of the causes are better known than the whole cause. For the cause of the whole is composed out of the causes of the parts, but it is necessary that the things to be composed are known before the whole compound is known" (*DCo* 1.6.2; *OL* 1:59).

Hobbes discerns three different species of analysis in mathematics, corresponding to the three ways in which he thinks the equality or inequality of geometric figures can be determined, namely analysis by motion, by indivisibles, and by the "powers of lines." The first of these is grounded in the nature of motion and is specifically concerned with the motions by which figures are produced, "for from motion and time the equality and inequality of any quantities can be argued, no less than by congruence; and some motion can be made so that two quantities, whether lines or surfaces, although one is straight and the other curved, are congruent in extent and coincide; which method Archimedes used in his treatise on spirals" (*DCo* 3.20.6; *OL* 1:254). Hobbes identifies the second sort of analysis with the method of indivisibles, since "equality and inequality are often found by the division of two quantities into parts which are considered as indivisible, as Bonaventura Cavalieri has done in our time, and Archimedes in the quadrature of the parabola" (*DCo* 3.20.6; *OL* 1:254). The final style of analysis "is performed by considering the powers of lines, or the roots of powers, by multiplication, division, addition, subtraction, and the extraction of roots from powers, or by finding where right lines terminate in the same ratio" (*DCo* 3.20.6; *OL* 1:254).

The first and third types of analysis are worth investigating somewhat more closely. When he first published *De Corpore,* Hobbes was convinced that he had found a new mathematical method, which he called analysis by motions or the method of motion.[34] He had high hopes for this new type of analysis, having convinced himself that it could deliver results vastly exceeding those available with traditional means. Wallis remarks that in the penultimate impression of chapter 20 Hobbes had claimed in article 6 that the construction used in his quadrature was an example of the "analysis by computation of motions"; this was later abandoned when Hobbes recognized the failure of this attempt to square the circle and the quadrature was amended to be stated "problematically."[35] Evidently confident that this new style of analysis was a fundamental advance, Hobbes had planned to announce:

> From those things which have been said concerning the dimension of the circle, so many others can be deduced, that it seems to me that geometry, which for the longest time has remained stalled in this place as if before the pillars of Hercules, is now thrust forth into the ocean, to navigate the globe of other most beautiful theorems. And indeed I do not doubt that from the known ratio of right lines to the arc of a circle, to the parabola, and to the spiral, knowledge will at last be threshed out *[extitura]* of the ratio of the same right line even to a hyperbolic line, an elliptical line, and even the discovery of any number of continually proportional means between two extremes. (*Elenchus* 129–30)

In the final impression of *De Corpore,* Hobbes's found himself forced to admit the inadequacy of his three circle-squaring efforts and to drop this exultant passage. Instead, he concluded section 20 with the mournful declaration of the quadrature's shortcomings, while justi-

34. He retains the term "method of motion" in the *Six Lessons,* notwithstanding the fact that he had had to give up many of his claimed results. In the fifth lesson he declares: "But because you pretend to the Demonstration of some of these Propositions [in *DCo* 3.17] by another Method in your *Arithmetica Infinitorum,* I shall first try whether you be able to defend those Demonstrations as well as I have done these of mine by the Method of Motion" (*SL* 5; *EW* 7:307).

35. Wallis writes: "Then, after you have further vilified the 'analytics by way of powers,' in order to commend your new 'analytics by the computation of motions' as more proper, you set forth an example of it (at least in the penultimate impression); that is, you show by which motions this analytics had applied to your first quadrature, which analysis we have examined above" (*Elenchus* 129).

fying his decision to leave his bitter remarks aimed at Ward (Vindex) unaltered:

> Since (after this was worked out [*excusa*]) I have come to think that there are some things that could be objected against this quadrature, it seems better to warn the reader of this fact than to delay the edition any further. It also seemed proper to let stand those things which are deservedly directed at Vindex. But the reader should take those things which are said to be found exactly of the dimension of the circle and of angles as instead said problematically. (*DCo* 3.20.8; 1655 edition only)

As I indicated in the third chapter, Hobbes's comments on his powerful new "analysis by motions" invite the conjecture that this method is related to Roberval's method of "composition of motions." Hobbes and Roberval were in contact during Hobbes's stay in Paris in the 1640s, they participated in discussions on mathematical matters at Mersenne's lodgings, and they were both active in the Parisian mathematical community. We also saw that they had a common interest in such problems as the rectification of curvilinear arcs, and particularly in the problem of comparing the arc length of the cycloid with that of the parabola. Roberval's principal innovation in the method of indivisibles was to employ a kinematic analysis in the solution of such problems. This approach involves treating curves as traced by the motions of points and then comparing the infinitesimal motions by which the curve is generated at any given instant. Pierre Costabel has remarked that Roberval's successful solutions to various geometric problems "were connected to a very new conception of curves, in which that construction by a point was given independently of the sort of algebraic analysis that Descartes used. . . . It is this notion that led him to a method of composition of motions that enabled him in 1640 to solve problems of determining tangents" (Costabel and Martinet 1986, 23–24). Whether, in fact, Hobbes's conception of the "analysis by motions" is directly related to Roberval's work remains unclear, since Hobbes's attempted quadrature of the circle contains relatively little reference to the generation of the circle by motion, and does not attempt to compare infinitesimal arcs with rectilinear motions.[36] For his

36. Bernhardt observes that "[i]n 1642, during the winter, Hobbes discussed mathematics with the geometer [Roberval] in Mersenne's lodgings. It was the question of the rectification of the arcs of the spiral of Archimedes and the parabola. The surviving documents allow the Roberval scholar Kotiki Hara to conclude that the philosopher without doubt put the geometer on the track of a kinematic demonstration that these

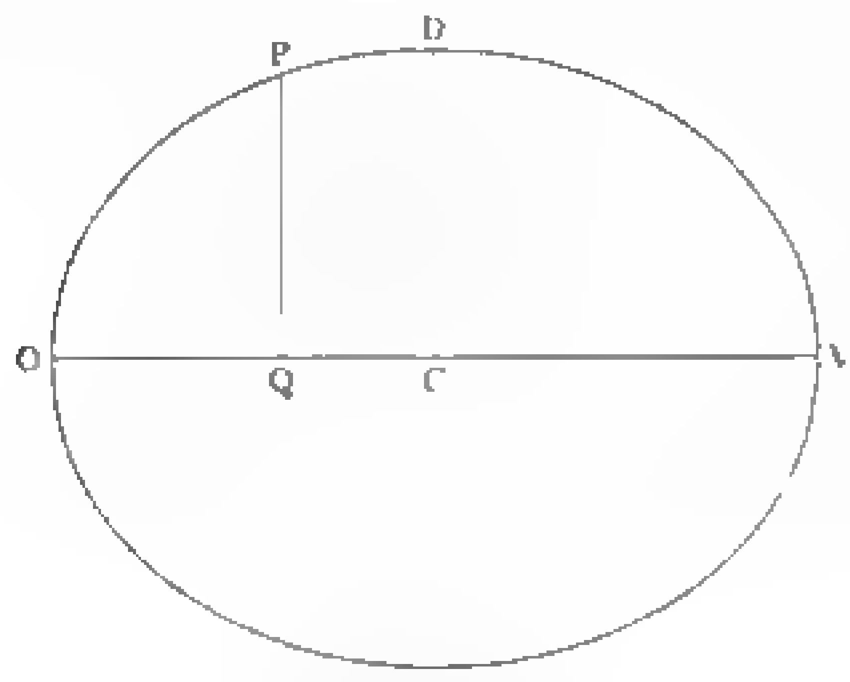

Figure 5.2

part, Wallis was convinced that Hobbes's talk of a new method of "analysis by motions" had been taken from Roberval, misunderstood by Hobbes, and ineptly inserted into the discussion of the circle quadrature.

Hobbes contrasts his new style of analysis to the more traditional analysis "by the power of lines," which is his way of characterizing the classical theory of loci. Consider, for example, the classical definition of the ellipse with major axis *OV* and center *C* (figure 5.2). Apollonius and the tradition define it as the locus of points satisfying the condition that the perpendiculars *PQ* and *DC* stand in the ratio $PQ^2/CD^2 = (OQ \cdot QV)/(QC \cdot CV)$. In Hobbes's estimation, "that part of analytics that is by powers, although it is regarded by some geometers (not of the first rank) as apt for the solution of all problems, yet it is a thing of no great extent" (*DCo* 3.20.6; *OL* 1:256). The limitations of this style of analysis arise from the fact that it "is all contained in the doctrine of rectangles, and rectangled solids. So that although they come to an equation that determines the quantity sought, yet they cannot sometimes by art exhibit that quantity in a plane, but in some conic section; that is, as geometricians say, not geometrically, but mechanically" (*DCo* 3.20.6; *OL* 1:256).

Although Hobbes sanctions both analytic and synthetic reasoning in geometry (and is particularly fond of his analysis by motions), he holds that only synthesis can be truly demonstrative. The reason for this should be clear: analysis proceeds hypothetically, but synthesis

discussions left incomplete" (Bernhardt 1989, 107–8). The reference to Kotiki Hara is to a doctoral dissertation defended at Paris in 1965, which I have unfortunately not been able to locate.

against what he takes to be their excessive use of symbols. In *De Corpore* he dismisses the techniques of analytic geometry as "the brachygraphy" (i.e., shorthand) of geometry, a term meant to capture the idea that symbolic methods may abbreviate proofs but cannot fundamentally increase the power of the traditional methods. Such methods, he says, are part of the "analysis by powers," and are not used by geometers "of the first class" (*DCo* 3.20.8; *OL* 1:256). In one especially unrestrained outburst in the *Six Lessons,* he attacks Wallis's use of algebra by asking

> When did you see any man but your selves publish his Demonstrations by signs not generally received, except it were not with intention to demonstrate, but to teach the use of Signes? Had *Pappus* no Analytiques? Or wanted he the wit to shorten his reckoning by Signes? Or has he not proceeded Analytically in a hundred Problems (especially in his seventh Book), and never used Symboles? Symboles are poor unhandsome (though necessary) scaffolds of Demonstration; and ought no more to appear in publique, then the most deformed necessary business which you do in your Chambers. (*SL* 3; *EW* 7:248)

Elsewhere in the *Six Lessons,* he remarks that Wallis's *Treatise of Conic Sections* "is so covered over with the scab of Symboles, that I had not the patience to examine whether it be well or ill demonstrated" (*SL* 5; *EW* 7:316). He ultimately finds in the mathematics of the Savilian professor "no knowledge neither of Quantity, nor of measure, nor of Proportion, nor of Time, nor of Motion, nor of any thing, but only of certain Characters, as if a Hen had been scraping there" (*SL* 5; *EW* 7:330). This much of Hobbes's case against algebraic methods is hardly worth taking seriously, since it amounts only to an "aesthetic" complaint that symbolic methods deface geometric demonstrations.

Hobbes adds a more interesting criticism when he contends that the introduction of algebra cannot add anything to geometry because it distracts the geometer's attention from geometric magnitudes and replaces the contemplation of magnitudes with the manipulation of symbols. This is the import of his accusation that Wallis is misled by empty symbols that have been added on to the old "analysis by powers" but have not produced anything new:

> I verily believe that since the beginning of the world, there has not been, nor ever shall be so much absurdity written in Geometry, as

> is to be found in those books of [Wallis]. . . . The cause whereof I imagine to be this, that he mistook the study of *Symboles* for the study of *Geometry*, and thought *Symbolicall* writing to be a new kinde of Method, and other mens Demonstrations set down in *Symboles* new Demonstrations. The way of *Analysis* by Squares, Cubes &c., is very antient, and usefull for the finding out whatsoever is contained in the nature and generation of rectangled Plains (which also may be found without it) and was at the highest in *Vieta;* but I never saw anything added thereby to the Science of Geometry, as being a way wherein men go round from the Equality of rectangled Plains to the Equality of Proportion, and thence again to the Equality of rectangled Plains; wherein the *Symboles* serve only to make men go faster about, as greater Winde to a Winde-mill. (*SL* dedication; *EW* 7:187–88)

Where others had taken algebra to be a way of simplifying demonstrations while also enabling geometers to find new results, Hobbes sees algebra as only a new kind of language that has been foisted upon geometry to no purpose.

In a similar vein, he claims that algebra cannot shorten demonstrations or make geometry easier to understand. Contrary to the claims of the modern analysts, Hobbes insists that "algebra can yield brevity in the writing of a demonstration, but not brevity of thought. Because it is not the bare characters, or only the words, but the things themselves that are the objects of thought, and these cannot be abbreviated" (*Examinatio* 3; *OL* 4:97). As Hobbes sees the matter, a proper demonstration must proceed by way of constructions from causes to effects, but reliance upon algebra simply interposes a collection of symbols between ourselves and the magnitudes we are to construct.

Hobbes's distinction between analytic and synthetic reasoning also accounts, in part, for his rejection of analytic geometry. A typical proof in analytic geometry proceeds "analytically" in the classical sense by first supposing the problem solved and then showing that the solution is algebraically admissible. As Descartes puts it:

> If, then, we wish to solve any problem, we first suppose the solution already to have been effected, and give names to all the lines that seem needful for its construction—to those that are unknown as well as to those that are known. Then, making no distinction between known and unknown lines, we must unravel the difficulty in any way that shows most naturally the relations between these lines, until we find it possible to express a single

> quantity in two ways. This will constitute an equation, since the terms of one of these two equations are together equal to the terms of the other. And we must find as many such equations as we have supposed lines which are unknown. (*AT* 6:372–73)

This haphazard approach is not a proper demonstration by causes and, Hobbes claims, can at best result in a half-finished demonstration that must still be "converted" into a synthesis. True geometry must appeal to causes and constructions, rather than the hypothetical procedure of Descartes and its fortuitous unraveling of a problem by means of equations.

We can thus discern two strands in Hobbes's objection to analytic geometry: he opposes both its excessive use of algebraic symbols and its reliance upon hypothetical procedures. These two strands of Hobbes's critique of analytic geometry can be drawn together and phrased in a single objection: the use of algebraic methods in geometry must be either unscientific or superfluous. For if algebraic techniques have governing principles and do not simply proceed *par hasard*, then these principles must be vindicated by appeal to geometric considerations, thereby making algebra superfluous. Hobbes poses this dilemma in the *Examinatio* as follows:

> What else do the great masters of the current symbolics, Oughtred and Descartes, teach, but that for a sought quantity we should take some letter from the alphabet, and then by *right* reasoning we should proceed to the consequence? But if this be an art, it would need to have been shown what this *right* reasoning is. Because they do not do this, the algebraists are known to begin sometimes with one supposition, sometimes with another, and to follow sometimes one path, and sometimes another. . . . Moreover, what proposition discovered by algebra does not depend upon Euclid (6, prop. 16) and (1, prop. 47), and other famous propositions, which one must first know before he can use the rules of algebra? Certainly, algebra needs geometry, but geometry does not need algebra. (*Examinatio* 1; *OL* 4:9–10)

Clearly, the "right" reasoning Hobbes has in mind here will be reasoning from causes to effects, that is to say demonstrative knowledge grounded in a synthetic exposition of the properties of geometric objects. Hobbes holds that we cannot know whether an algebraic procedure is legitimate unless we already know that the geometrical step corresponding to it is an admissible construction. But in such a case

Rather, it is a way to place geometric problems in a very general setting and (as Wallis would put it) to remove any specifically geometric content while demonstrating properties of the curves from the algebraic features of their characteristic equations.[41] The fruitfulness of analytic geometry is manifest: with it, mathematicians solved important problems left unsolved by classical methods while opening up whole new classes of problems that could never be attempted within the confines of the classical point of view. Hobbes wanted desperately to make his mark as a mathematician, but his own theory of demonstration encouraged him to toss aside the very tools that might have helped him achieve his dream.

41. This is the "emphasis on the relations rather than the objects" and the "freedom from ontological commitment" that Mahoney (1980) identifies as essential to the rise of algebraic thought and in large measure responsible for the mathematical successes enabled by algebraic methods.

CHAPTER SIX

The Demise of Hobbesian Geometry

> I do not want to change, confirm, or argue any more about the demonstration which is in the press. It is correct; and if people burdened with prejudice fail to read it carefully enough, that is their fault, not mine. They are a boastful, backbiting sort of people; when they have built false constructions on other people's principles (which are either false or misunderstood), their minds become filled with vanity and will not admit any new truth.
>
> —Hobbes to Sorbière, 7/17 March 1664

Although there may be many disputes in which it is misleading to speak of winners and losers, there is no question that, at least in regard to its mathematical aspects, Wallis was the winner in his dispute with Hobbes. He exposed the inadequacy of more than a dozen Hobbesian circle quadratures and soundly refuted many other of Hobbes's excursions into the great classical problems. Indeed, he was so successful in his campaign against Hobbes that the only blemish to his own reputation was essentially self-inflicted: contemporaries such as Huygens readily granted the cogency of his arguments but wondered why he had taken the trouble to expose the vanity of Hobbes's pretenses at such great length.[1] Well before his death in 1679, Hobbes's once-considerable mathematical reputation had been utterly destroyed and, although Wallis had hardly enhanced his standing in the course of the dispute, it was evident to any seventeenth-century observer that he had defeated his antagonist in their very publicly conducted contest.

1. Commenting on the *Elenchus* in a letter to Wallis dated 5/15 March 1656, Huygens remarks that "I was amazed that you judged [Hobbes] worthy of such a lengthy refutation, although I read your learned and rather sharp *Elenchus* with some pleasure" (*HOC* 1:392). Oblivious to the suggestion that he might have found a better and more dignified use for his time, Wallis did not hesitate to publish Huygens's comments in order to show the world how poorly Hobbes's efforts had fared with the learned public (*Due Correction* 4–5).

This is not to say that Hobbes had absolutely no allies, but there is hardly anything that one might term a discernibly Hobbesian school of mathematics in seventeenth-century intellectual life.[2]

My purpose in this chapter is to examine Hobbes's increasingly desperate attempts to defend the validity of his mathematical work and thereby to examine the process by which he was led to reject ever-larger portions of geometry in order (so he thought) to shield his claims from refutation. This process went so far that by the end of his life Hobbes found it necessary to condemn essentially all of classical as well as contemporary geometry as ill conceived. Being driven to such an expedient by the continued refutation of his work must certainly have been an unpleasant fate for the man who had once declared classical geometry "the onely Science it hath pleased God hitherto to bestow on mankind" (*L* 1.4, 15; *EW* 3:23). There is much to learn about Hobbes by studying his seemingly forced march toward the abandonment of accepted mathematical practices and results, and we can gain important insights into his philosophy as a whole by attending to some nuances in his changing attitude toward traditional mathematics.

2. Although there was no Hobbesian school of geometry, Hobbes's mathematics was not universally condemned. François du Verdus was generally receptive to Hobbes's mathematical efforts, and Hobbes repaid his mathematical loyalty by dedicating *Examinatio et Emendatio Mathematicae Hodiernae* to him. Eventually du Verdus advised Hobbes to abandon the search for the circle quadrature and admit error, since "you will still have the glory of having penetrated as far as anyone else in that matter—and further than anyone else in every other matter" (Du Verdus to Hobbes 22 December 1656/1 January 1657; *CTH* 1:413). François Pelau, another French admirer of Hobbes, held his mathematical work in even higher esteem. In a letter to Hobbes in May of 1656, Pelau wrote that "it is because of you that I have turned to the truth, having rebelled against it in favor of the Ancients; your section *De Corpore* is a work which will live and be read and admired by the most distant future generations. I was extremely glad to see that you are a great geometer: it was in that capacity, above all, that you touched on my own interests. I have seen the works of Descartes, Gassendi, Galileo, and Mersenne, but they all amount to nothing in comparison with what I learn every day from your book, which I have so thoroughly assimilated that I hardly read anything else: it alone is my entire library, and the subject of all my commentaries" (Pelau to Hobbes, 18/28 May 1656; *CTH* 1:291). Other reports of Hobbes's mathematical supporters are hard to come by. Stubbe reported to Hobbes in early 1657 that admirers in Oxford had composed verses on the battle between Hobbes and Wallis, so that "yo^u^ may see y^e^ vogue of those youths that pretend to any thing of ingenuity is against D:^r^ Wallis, & yo^u^ have the good opinion of all who are judges of language, ingenuity or Mathematiques" (Stubbe to Hobbes, 30 January/9 February 1657; *CTH* 1:440). After Hobbes's death, Venterus Mandey translated some of Hobbes's mathematical works into English and evidently found them well suited for teaching such practical subjects as measurement and surveying (Mandey 1682). Still, none of this amounts to a Hobbesian mathematical school.

6.1 THE DESCENT INTO THE ABYSS

I argued in chapter 3 that there is an identifiable core of doctrines that comprise the Hobbesian philosophy of mathematics, the most important of which are the claims that (1) mathematics is a generalized science of body and (2) the first principles of mathematics must express the causes by which mathematical objects are generated. Although Hobbes never abandoned these core principles, his views on the nature of mathematics were not altogether static. In the period after the publication of *De Corpore* (1655) he changed his mind on several significant mathematical issues, and particularly about the extent to which classical geometric methods could be reconciled with his own materialistic program for mathematics. Hobbes was soon forced to face the unpleasant fact that his supposed solutions to classical problems were refuted by Wallis and other European mathematicians. His initial reaction to such refutations was generally to admit error on points of technical detail while insisting that his general conception of mathematics was the only coherent philosophical account of the subject. However, as the refutations mounted and his revised efforts fared as badly as the originals, Hobbes began to challenge the validity of criticisms and to reject the principles upon which they were based. In the end, this process went so far that he abandoned even the most elementary principles of classical geometry and recanted his earlier admissions of error.[3]

I have chosen to illustrate the collapse of Hobbes's geometric program by an examination of his responses to criticisms of his proofs and document the increasingly desperate measures he undertook in defense of his claims. I begin with an account of his replies to technical objections to the geometry of *De Corpore*, proceed to an examination of his unsuccessful efforts to duplicate the cube in the 1660s, then turn to an overview of his essay *De Principiis et Ratiocinatione Geometrarum*, and conclude with a brief study of his attempted quadrature of the circle from 1669. This selection, although necessarily incomplete, gives an adequate picture of the difficulties Hobbes encountered and of the increasing desperation of his responses. Along the way, I

3. On Hobbes's recantation of prior admissions of error, Wallis observed with characteristic venom, "How many Quadratures, first and last, Mr *Hobs* hath furnisht us with; I cannot presently tell You. But that they are all true, and all the same, I suppose he would have us beleeve. For though he have formerly confessed some of them to be mistakes; yet he hath now revoked those confessions, and thinks them to be true" (*HHT* 104).

will take the opportunity to study Hobbes's failed campaign for admission to the Royal Society and its connection with his geometric work.

6.1.1 Hobbes in Defense of De Corpore

I showed in chapter 3 that the circle quadrature in section 20 of *De Corpore* has a tortured history that reflects Hobbes's apparent indecision about the soundness of his argumentation. He assembled several different versions of this result, replacing versions one after another as friends made him aware of their shortcomings. In the end, the text of *De Corpore* as Wallis encountered it had a decidedly "pentimento" character that made the refutation and humiliation of Hobbes all the easier. Notwithstanding the acknowledged errors in his first two attempts to square the circle, Hobbes insisted on publishing his original effort under the title "A false quadrature, from a false supposition," and downgraded the second to the status of an approximation, and finally had to admit that the third must be taken "problematically" rather than as a strict demonstration. All of this shows that Hobbes was prepared to recognize potential shortcomings in his geometrical proofs at the time he published *De Corpore*, and it is clear that he was by no means impervious to strictly technical criticisms of his work. He was convinced that he had squared the circle and was more than ready to revel in this imaginary triumph, but he was not irrevocably committed to any one particular solution and he was prepared to reconsider his efforts in the light of objections.

Hobbes's response to Wallis's *Elenchus* shows the same willingness to acknowledge error, despite the fact that his opponent's mocking tone and generally uncivil language gave him reason for resentment. In numerous passages from the *Six Lessons* Hobbes admits the inadequacy of his presentation and argumentation, although he frequently complains about the manner in which Wallis states his objections.[4] We can consequently take it as established that, as of 1656, Hobbes was

4. Thus, in the third of the *Six Lessons* (*EW* 7:267) he admits that the definition of geometric figure at *DCo* 2.14.22 "wants the same word" missing from the earlier definition of parallels at *DCo* 2.14.12 in order to make it completely general. Similarly, at the beginning of the fourth lesson he confesses that *DCo* 2.16 contains "three or four faults, such as any Geometrician may see proceed not from ignorance of the Subject, or from want of the Art of Demonstration, (and such as any man might have mended himself) but from security" (*SL* 4; *EW* 7:269). He further admits that a projected twentieth article to the sixteenth chapter of *DCo* contains "a great error," although the article was left out of the final version (*SL* 4; *EW* 7:297). Finally, he grants that in both of the problems that were supposed to be solved in the eighteenth chapter, "You have truly demonstrated that they are both false; and another hath also Demonstrated the same

ready to accept the authority of classical geometry and to submit his putative demonstrations to the tribunal of prevailing mathematical opinion.

Some examples drawn from the *Six Lessons* illustrate this claim more fully. In reply to Wallis's objections against the (admittedly false) first attempt to square the circle, Hobbes remarks that "seeing you knew I had rejected that Proposition, it was but a poor Ambition to take wing as you thought to do, like Beetles from my egestions" (*SL* 5; *EW* 7:324). Hobbes's point here is that the errors in his initial assault on the problem had already been acknowledged in the final printed text of *De Corpore*, so that the "false quadrature from a false supposition" need not be taken to show mathematical ineptitude on his part. Instead, he thought that by publishing an erroneous result and also showing the error in his procedure he would enable (or at least help promote) a true solution to the problem since "it was likely to give occasion to ingenious men (the practice of it being so accurate to sense) to inquire wherein the Fallacy did consist" (*SL* 5; *EW* 7:324). Wallis was hardly impressed by this display and commented that it was a sign of Hobbes's vanity and self-conceit, since he judged even his miserable failures to merit the attention of the learned world.[5]

Apparently unable to resist the temptation to make yet another foray into the treacherous terrain of circle quadrature, Hobbes even tried to counter Wallis's objections on a particular technical point.[6] But even here his language falls far short of a dogmatic assertion of the validity of his procedures. After attempting to rebut one of Wallis's criticisms of the construction and argumentation in *De Corpore*, Hobbes declares, "And though in this also I should have erred, yet it cannot be denied but that I have used a more natural, a more Geometrical, and a more perspicuous method in the search of this so difficult a Probleme, then you have done in your *Arithmetica Infinitorum*" (*SL* 5; *EW* 7:326). This is obviously not the language of a man who is completely confident of his results. Hobbes claims the superiority of his methods for being more geometrical and perspicuous, but he is

another way" (*SL* 5; *EW* 7:319). As we will see, the reference to "another" here is to Claude Mylon.

5. Before commenting on the failed quadratures in chapter 20 of *De Corpore*, Wallis remarked: "You do admit that both [of these quadratures] are false, but you bother to submit them, however false, to our view, obviously having judged that even in your miscarriages there is something of beauty" (*Elenchus* 90).

6. Wallis had argued that Hobbes's construction of a specific line was not equal to a given circular arc (*Elenchus* 108–11). Hobbes's argumentation and its flaws are examined in the appendix, section A.3.1.

willing to accept the possibility of error and does not claim that other approaches are entirely without merit.

This willingness to admit error arises, at least in part, from Hobbes's distinction between two fundamentally different causes of error in demonstration: ignorance and negligence. Errors of ignorance proceed from false or misunderstood principles, while errors of negligence involve the misapplication of correct principles. Errors of negligence are, in Hobbes's estimation, much less reprehensible than those of ignorance. The person "ignorant of that he goes about" commits a serious error "because he was not forced to undergo a greater charge than he could carry through" (*SL* 2; *EW* 7:212). Thus, an error of ignorance in a geometric demonstration shows that the geometer does not really understand the basis of his subject. In contrast, errors of negligence arise "through humane frailty" when true principles are mistakenly applied, the mistake being the consequence of the demonstrator "being less awake, more troubled with other thoughts, or more in haste when he was in writing" (*SL* 2; *EW* 7:212). This kind of error is naturally less reprehensible, first because it is common to all men, but more importantly because it does not reflect a shortcoming in the demonstrator's first principles.

Hobbes's admissions of error in *De Corpore* are confined exclusively to errors of negligence.[7] He remained adamant that his fundamental principles were above reproach, since he regarded his materialistic program for the philosophy of mathematics as the only proper way to understand such key concepts as equality, proportion, angle, point, line, figure, and measure. In other words, Hobbes placed more emphasis on the superiority of his overall philosophy of mathematics than on his ability to solve outstanding problems. He consequently did not regard it as particularly damaging if he should misapply one of his principles in the attempt to solve a problem.[8]

His willingness to acknowledge the validity of purely technical crit-

7. As I explain below, there is one apparent exception to this claim, when Hobbes admits that his general definition of parallels is incorrect. This might seem to fit his definition of an error of ignorance, as it involves a false or inadequate definition; but even here Hobbes insists that the difficulty is one of negligence because it is little more than a slip of the pen. Of course, from the fact that Hobbes only admits to errors of negligence, it does not follow that his errors are always such. In fact, as an inspection of his demonstrations readily shows, Hobbes frequently based his claims on false or misunderstood geometric principles.

8. In this context it is worth recalling Hobbes's boast to Wallis that "I am the first that hath made the grounds of geometry firm and coherent. Whether I have added anything to the edifice or not, I leave to be judged by the readers" (*SL* 3; *EW* 7:242).

icisms of his geometric work in *De Corpore* can also be seen in Hobbes's reaction to the objections of the French mathematician Claude Mylon.[9] Through the agency of Hobbes's friend du Verdus, Mylon sent Hobbes several objections to the mathematics of *De Corpore*. In particular, he charged that Hobbes's general definition of parallels (*DCo* 2.14.12; Hobbes 1655, 113) was incomplete, and that a consequence drawn from it did not follow.[10] He further objected to all of section 18 of *De Corpore* on the grounds that "the general problem that he uses to equate straight lines with parabolas would be fine if it were true; but the trouble is, M. Huygens of Zuylichem has demonstrated that it is false" (Mylon to du Verdus, enclosure in du Verdus to Hobbes 20 February/1 March 1656; *CTH* 1:241). Mylon added a demonstration of the falsehood of Hobbes's approach to the rectification of parabolic segments, basing this refutation on Huygens's treatise *De Circuli Magnitudine Inventa*.[11] One odd irony of this encounter between Huygens and Hobbes is that it apparently led Huygens to a

9. For an account of Mylon's life and work, see Noel Malcolm's brief biographical note in *CTH* 2:868–69.

10. The original definition reads, "Parallelarum rectarum definitionem aliquam habemus apud Euclidem, sed Parallelarum in universum definitionem nusquam invenio. Itaque earum Definitio Universalis esto haec. Duae *lineae quaecunque* (sive rectae, sive curvae) atque etiam duae superficies, *Parallelae sunt, in quas duae lineae rectae ubicunque incidenties, facientesque cum utravis ipsarum angulos aequales, sunt inter se æquales.*" The corrected version adds "ad easdem partes" between "ipsarum" and "angulos" (Hobbes 1655, 113; *OL* 1:163). The point is that two lines are parallel when any two right lines that intersect both and make equal angles on corresponding sides also have the same length (*SL* 3; *EW* 7:254). Du Verdus actually suggested the appropriate change to Hobbes, rewriting the original definition to read, "Any two lines whatsoever, straight or curved, and likewise any two surfaces, are parallel, when any two straight lines between them, intersected anywhere, make equal angles and are of equal length" (Du Verdus to Hobbes 20 February/1 March 1656; *CTH* 1:238).

11. The refutation is in the second of two enclosures in a letter from du Verdus to Hobbes, and may have been the work of Huygens himself. We know that Huygens saw and refuted the quadrature in *De Corpore*, since Mylon refers to Huygens's own "examination" of Hobbes's "first construction" in a letter to Huygens (Mylon to Huygens, 13/23 June 1656; *HOC* 1:440). Furthermore, Mylon says in his letter to du Verdus that contains the enclosures that Huygens "is due to write about this from Holland, to Mr Hobbes. If you are curious to see this demonstration of his, I shall send it to you whenever you like" (Mylon to du Verdus 20 February/1 March 1656; *CTH* 1:241). Malcolm reports that both enclosures are in Mylon's hand and attributes both to Mylon (*CTH* 1:240). Whether or not Huygens actually penned the refutation, it remains the case that the real source of the objection are the results in his *De Circuli Magnitudine Inventa*. This treatise can be found in *HOC* 12:91–181. Mylon sent a second version of the argument to Hobbes in April of 1656, again basing his argument on proposition 9 of Huygens's *De Circuli Magnitudine Inventa* (Mylon to Hobbes, 9/19 April 1656; *CTH* 1:272–76).

close examination of the problem of rectifying parabolic arcs, which eventually produced a correct result.[12]

Hobbes's response to Mylon's criticisms was to concede error. With evident reluctance, he admitted that his definition of parallels was inadequate and corrected it in the English version of *De Corpore*. He nevertheless took great pains to deny that his mistaken definition constitutes an error of ignorance, precisely because he could not bring himself to admit error on a point of general principle. In the *Six Lessons* Hobbes tried both to downplay the significance of the erroneous definition of parallels and to contrast Mylon's manners with those of Wallis:

> At the twelfth Article, I confess your exception to my universal definition of Parallels to be just, though insolently set down. For it is no fault of ignorance (though it also infect the demonstration next it) but of too much security. . . . The same was observed also upon this place by one of the prime Geometricians of *Paris*, and noted in a Letter to his friend in these words, *Chap.* 14. *Art.* 12. *the Definition of Parallels wanteth somewhat to be supplyed.* And of the Consectary, he says, *it concludeth not, because it is grounded on the Definition of Parallels.* Truely, and severely enough, though without any such words as savour of Arrogance, or of Malice, or of the Clown. (*SL* 3; *EW* 7:255)

It is noteworthy that even in this case Hobbes insists that his error does not involve ignorance, notwithstanding the fact that it concerns a general principle. Given Hobbes's penchant for reprimanding his opponents for their failure to proceed from proper definitions, this is not the kind of error he could cheerfully admit. Thus, rather than acknowledge an error of ignorance on this point, Hobbes characterizes his mistake as a simple slip arising from "too much security"—which is his way of saying that it proceeds from inattention to detail, presumably

12. "Occasioned by the refutation of Thomas Hobbes's failed rectification of the parabola, Huygens showed that by employing a system of chords between equidistant parallels to the diameter (which could then be transformed into a system of tangents), the arc of a parabola could be straightened, and he then reduced the rectification to the quadrature of the hyperbola" (Hoffmann 1957, 2:39). Whiteside mentions that Hobbes "tried to show that the general parabola-arc is equal to a rational line-length, not allowing unfortunately for the modifying effect of changing gradient" (1960–62, 328). Huygens first pointed out Hobbes's error in a letter to Wallis from 5/15 March 1656 (*HOC* 1:392), but his solution to the problem was not worked out until 17/27 October 1657 (see *HOC* 14:188–92, 234–36).

De Corpore show him to have been generally engaged in the mathematics of his day in the 1650s. Although his grand designs did not meet with the approval for which he had hoped, Hobbes still accepted the same basic framework as his critics and he was ready to have his work judged by the standards of demonstration that prevailed throughout the European mathematical community. There is consequently no reason to imagine some kind of "radical incommensurability" between Hobbes and his critics at this point in his career, precisely because Hobbes both understood the objections raised to his work and attempted to make his proofs conform to the recognized standards of geometric demonstration. As it happens, even his amended efforts were unsuccessful, but even this does not show that there is some kind of barrier to mutual understanding between Hobbes on the one side and Wallis, Mylon, or Huygens on the other. All the players in the dispute understood each other's claims and agreed upon the standards by which they should be judged. However, Hobbes's acceptance of his opponents' standards of proof was not permanent. As we will see, his defense of his attempted duplication of the cube led him to reject essentially all of classical mathematics.

6.1.2 The Beginning of the End: Hobbes on the Duplication of the Cube

Hobbes's confidence in his mathematical abilities remained high in the early 1660s, but he was less willing to accept the authority of traditional geometric opinion in determining the validity of his results. This fact is best illustrated by an analysis of his 1661 duplication of the cube and his response to criticisms of it. Where he had earlier been prepared to admit error, Hobbes had by this time become less willing to acknowledge even technical shortcomings in his demonstration, and he contested the applicability of algebraic arguments against his geometric constructions. He pursued this resistance even to the point of rejecting the most basic principles of mathematics, finally abandoning the Pythagorean theorem and the entire classical theory of magnitudes in order to preserve his alleged results from algebraic refutations. It is worthwhile to follow this process in some detail. By considering his replies to various criticisms of his mathematics, we can see that Hobbes did not make a sharp break with classical geometry, but instead saw himself forced to adopt ever more desperate measures to defend the integrity and validity of his (alleged) solutions to the great classical problems.

Hobbes published *La Duplication du Cube par V.A.Q.R.* anonymously in Paris in 1661.[16] The reasons for his choice of anonymous publication in a foreign language and in a foreign country are unclear. Robertson claims that he intended "to put Wallis and other critics off the scent, and extort the judgement that apparently was now to be denied to any mathematical work of his" ([1806] 1910, 179), and Scott repeats this opinion almost verbatim (1938, 167). Although Hobbes would later make no attempt to hide his authorship of this or any other of his mathematical pieces, his own remarks on the cube duplication lend some support to Robertson's opinion.[17] Whatever the reason for the peculiar manner of its publication, Wallis immediately recognized Hobbes's handiwork and refuted it in a letter dated 23 June/3 July 1661.[18] To my knowledge the only surviving copy of the piece is item 85 among the Hobbes manuscripts in the library at Chatsworth House. This is a single sheet printed on both sides, although it

16. The expression "V.A.Q.R.," Hobbes later tells us, was meant to stand for "Un Autre Que Roberval" (*DP; OL* 4:295).

17. In particular, Hobbes's *Seven Philosophical Problems* contains a dialogue on the problem, in which the interlocutor A asks, "Have you seen a printed paper sent from Paris, containing the duplication of the cube, written in French?"; to which the intrepid B replies, "Yes. It was I that writ it, and sent it thither to be printed, on purpose to see what objections would be made to it by our professors of algebra" (*SPP* 8; *EW* 7:59). Wallis was the first to conjecture that Hobbes's choice of a French publication was a ruse designed to confuse his opponents:

> Observing that *Mr Hobs's Geometry* (whether by reason of others Envy, or for what other cause, *I* will not now dispute) was not now in any great Repute; and, fearing least that *Odium Hobii*, which he so much complaines of, as so prejudicial to Man-kind in hindring the reception of his Notions, without which it is impossible to make any progresse in the Search of Nature; . . . might be prejudicial also to this of the *Cube*, (and, thereby, not onely deprive him of the Credit, but all man-kind of the Benefit, of his New Discovery:) To obviate those evils; he caused his *Probleme* of *Doubling the Cube*, to be printed in *French*; . . . and divers papers of it to be given abroad, which were pretended to be brought from *Paris;* (For had it been in *English*, or thought to be done at home, the Matter would presently have betrayed the Author:) Not doubting, but that, the *Odium* would cease to operate when the Person was concealed; and, no *Prejudice* obstructing an impartial Estimate, his Demonstration would presently find Reception and Approbation: Which could not afterwards be withdrawn, when He should appear to be the Author. (*HHT* 127–28)

18. This letter is contained among Hobbes's papers in at Chatsworth House (Hobbes MSS., letter 51). Malcolm has shown that is unlikely to have been addressed to Hobbes because it refers to him in the third person and uses terms of address more suited to a nobleman, probably Viscount Brouncker (*CTH* 1:xlvi). In any event, the letter must have been forwarded to Hobbes.

mained Hobbes's claimed solution to the problem for the rest of his life.[20]

The details of Hobbes's argument can be found in section A.2 of the appendix and will be left aside here. It is important for the present context to note that Hobbes's construction succeeds only if he can establish that the line *YZ*, when extended, meets *DP* at *P*. It should come as no surprise to learn that the construction fails to solve the problem, notwithstanding the detailed construction and convoluted argument he adduces for the result.[21] The inadequacy of the argument can be gleaned from the objections raised by Wallis in his letter of 23 June/3 July 1661. Wallis's refutation of the *Duplication du Cube* comes in three parts: first a purely geometric argument to show that *YZ* extended does not meet *DP* at *P*; second the observation that, because the demonstration makes no use of the length of the line *DP*, it could be assigned any length whatever and hence the demonstration fails to determine that *YZ* extended intersects *DP* at *P*; and third an essentially algebraic reductio ad absurdum argument showing that the original construction cannot yield lines *DY* and *DX* exhibiting the desired proportionality. The first two arguments are not sufficiently interesting to detain us,[22] but it is worthwhile to consider the third, as Hobbes's reaction to it is quite significant.

20. It can be found with inessential variants in the *Dialogus Physicus* of 1661 and the English translation of that work in *Seven Philosophical Problems* from the same year (although this was not actually published until 1682, after Hobbes's death). One characteristic of Hobbes's original construction that I have left out of this summary is the superfluity of many of the lines and arcs he constructs. As can be easily seen from an examination of the original version given in the appendix, many of the lines Hobbes inserts in the construction are irrelevant to his argument. Wallis drew attention to this feature of Hobbesian mathematics when he remarked, "You may perhaps wonder (and so did I till I knew Mr *Hobs* was the Author of that Paper) why he should clog his *Figure*, and the *Construction of it*, with such a Multitude of superfluous Lines and Letters, whereof he makes not use at all either in the *Construction of the Probleme* or the *Demonstration* of that Construction" (*HHT* 132). As his cube duplication went through several versions, Hobbes did delete some of these extraneous lines, but the essentials of the construction did not change.

21. This part of Hobbes's cube duplication underwent a number of changes, as he sought arguments to show that the prior construction did, in fact, yield a line YZ that, when extended, meets DP at P.

22. In the first of these objections, Wallis shows that the construction and demonstration used by Hobbes only license the inference that a particular figure in the construction is a rectangle, not a square as Hobbes believed. Wallis examines the matter at greater length in *HHT* 134–44. The second objection (although obviously fatal to Hobbes's demonstration) seems to have had no effect on Hobbes, who simply dismisses it as the work of a man with no grasp of logic or demonstration.

mately led Hobbes to his most radical (indeed, incoherent) dissent from the mathematics of the seventeenth century.

Hobbes links this doctrine to his general philosophy of mathematics, arguing that because the "algebraists" are accustomed to treat lines as lacking breadth, their calculations must fail to account for the small overlap between two lines in a geometric figure. In the *Dialogus Physicus*, one of the interlocutors asks "although the arithmetical calculation differs from the geometric one, why does it differ by so little, namely as much as the difference between $\sqrt{1681}$ and $\sqrt{1682}$?" The answer is quite remarkable: "Because whoever multiplies lines considered as without latitude does not make a plane, but a number of lines. But whoever draws one right line into another right line does not make a number of lines, but a plane surface. Thence it necessarily happens that in the sides of the planes the points, which are in the common angles of two right lines, are counted twice, and in the sides of cubes three times" (*DP; OL* 4:294–95).

At this stage in his career (i.e., the early 1660s), Hobbes was clearly opposed to the prevailing view of geometry, but his resistance to the authority of classical mathematics was not yet absolute. He continued to seek the opinions of recognized mathematicians (excepting Wallis) in confirmation of his duplication of the cube, and his attitude could hardly be characterized as a dogmatic dismissal of the mathematical principles and practices of his day. John Pell, whom Hobbes had assisted in the campaign against the circle-squaring efforts of Longomontanus nearly twenty years earlier, recorded a meeting with Hobbes on 31 March/10 April 1662, as Hobbes was preparing his *Problemata Physica* for the press. Pell represents the aged sage of Malmesbury as convinced of the validity of his result but still concerned to gain the approval of the mathematical world:

> This morning M^r^. Thomas Hobbes met me in the Strand, & led me back to Salisbury house, where he brought me into his chamber, and there shewed me his Construction of that Probleme, which he said he had solved, namely *The Doubling of a Cube*. He then told me, that Viscount Brounker was writing against him. But, said he, I have written a Confirmation & Illustration of my Demonstration; and to morrow I intend to send it to the presse, that with the next opportunity I may send printed coppies to transmarine Mathematicians, craving their censure of it. On this side of the sea, said he, I shall hope to have your approbation of it. I answered, that I was then busy, and could not per-

> swade my selfe to pronounce of any such question, before I had very thoroughly considered it, at leysure, in my owne chamber. Where-upon he gave me these two papers, bidding me take as much time as I pleased. Well, said I, if your work seeme true to mee, I shall not be afraid to tell the *world* so: But if I finde it false, you will be content that I tell *you* so: But privately, seeing you have onely thus privately desired my opinion of it. Yes, said he, I shall be content, and thanke you too. But, I pray you, doe not dispute against my Construction, but shew me the fault of my Demonstration if you finde any. Thus we then parted, I leaving him at Salisbury house, and returning home. (British Library MS. Add. 4425, f. 238r)

The "two papers" to which Pell refers in this account are presumably a Latin version of Hobbes's circle quadrature and its accompanying diagram (British Library MS. Add. 4225, f. 215, f. 217). Pell obliged by refuting the duplication and leaving a copy of the refutation with Hobbes, to which Hobbes evidently responded by offering a new argument to show that his construction succeeded.[26]

The significance of this account is great, for it shows something of the state of Hobbes's mathematical methodology in the spring of 1662. He was then still prepared to submit his work to the judgment of recognized mathematicians, but he was clearly less open to criticism than he had been. Hobbes's insistence here that Pell "not dispute against my Construction, but shew me the fault of my Demonstration" is an interesting restriction. The demand shows that Hobbes was not completely prepared to accept all of the principles of traditional mathematics, although he certainly saw value in some criticisms from the classical standpoint. A geometric proof (or attempted proof) such as Hobbes's *Duplication du Cube* traditionally contains two sections: the construction and the demonstration.[27] The construction is typically stated in the imperative mood and bids the reader effect certain constructions, such as "take a line of length AB in the diagonal CD," or "divide the arc SQ into two equal parts." The demonstrative phase argues that the lines and figures thus constructed have certain properties. This section of the proposition thus asserts, for example, that "the line

26. A copy of Pell's refutation survives as British Library MS. Add. 4425, f. 216, which contains Pell's note: "I left a copy of this with M^r Hobbes May 5." Hobbes's reply is MS. Add. 4425, f. 236 (a diagram) and f. 237 (a revised argument, with a note from Hobbes to Pell).

27. See Heath's discussion of the formal divisions of a geometric proposition in Euclid [1925] 1956, 1:129.

BV is a mean proportional between the lines AD and DF," basing the assertion on specific features of the construction. There are, obviously enough, constraints on what can be constructed or demonstrated. The demonstration must avoid errors of logic and the construction must contain only operations permitted by the postulates of the geometric system. In the case we are considering, the construction must use only the familiar "compass and rule" operations characteristic of Euclidean geometry. By insisting that Pell confine his criticisms to the demonstration in the duplication of the cube, Hobbes in effect declared that his construction was immune to challenge and that he was prepared only to consider challenges to his efforts to show that the construction is a successful duplication of the cube. In part, this is due to Hobbes's insistence upon the claim that a properly developed geometry will use definitions that show how to construct the relevant objects. If the construction is in error, then there is reason to fear that it proceeds from false or inadequately understood first principles, which would be a much more serious sort of error.[28]

From the evidence of Pell's report, it is clear that Hobbes had high hopes for his duplication of the cube and its accompanying "confirmation and illustration." In particular, he had convinced himself that Continental mathematicians would welcome his latest excursion into the great classical problems, and he appears to have sent copies of this effort to his friend Samuel Sorbière in Paris with instructions to show them to leading mathematicians and gather their responses.[29] The *Problemata Physica* appeared in 1662 and consists of seven dialogues on problems in natural philosophy along with two mathematical appendices. The first appendix contains sixteen propositions on the quadrature of the circle, while the second is an expanded version of the cube duplication from 1661, along with a revised version of his replies to the objections of Wallis and Rooke.[30] It also contains an un-

28. Bird makes a similar point when he remarks that the distinction between construction and argumentation "allowed Hobbes tacitly to accept and correct fallacies in his argument while sticking to his quadrature and duplication claims. These were founded on the unchanged constructions, which, of course, embodied the all important generation of the desired quantities. The constructions, independently of the proofs, were embodiments of Hobbes's generative principle" (Bird 1996, 230).

29. It is not certain that Sorbière was Hobbes's intermediary in this project, but the letters between the two in the early 1660s do show that Sorbière was aware of Hobbes's mathematical struggles and that he served as his contact for many exchanges with French mathematicians.

30. The construction in the *Problemata Physica* differs minimally from that in the original *Duplication du Cube* of 1661 and the *Dialogus Physicus* of the same year. There

usual appeal "to foreign geometers" placed immediately after the dedicatory epistle and before the first chapter. In it, Hobbes declares that

> to these physical problems, which aim at nothing higher than verisimilitude, I have added some geometrical propositions that the professors of geometry and many others of our mathematicians do not accept, but think they have confuted by their arithmetical calculations. I alone (at least so far) contend against them that this arithmetic they use is suitable neither for the confirmation nor for the confutation of geometrical claims. I now appeal to you (readers of mathematics who have not yet condemned my works by prejudicial decree) to turn your faculties of reason to this discrepancy between my geometrical calculations and their arithmetical ones and (in the interest of mathematics itself) most humbly beseech and implore you to approve whichever one appears true. (Hobbes 1662a, sig. A8)[31]

This appeal was supplemented by a pronouncement at the end of the mathematical appendices to *Problemata Physica* that promised to end the controversy once and for all:

> If the geometers into whose hands these things come deem their opinions of these geometric computations worthy of sending to the bookseller whose name and address are to be found on the first page, then in the future either my antagonists will be silent, or I shall be silent. (Hobbes 1662a, 139)

In the course of events Hobbes saw fit to remove from subsequent versions of the *Problemata Physica* both his appeal to Continental mathematicians and his offer to remain silent. The reason for this apparent change of heart is not difficult to discern: his mathematics found no

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is consequently no need to investigate them in detail. The circle quadrature does differ from the earlier efforts in *De Corpore*, although an account of these differences is not of concern here. The reply to Rooke no longer uses an argument based on the computational error of taking $(\sqrt{2})^3 = 2$, but otherwise keeps the main line of analysis intact.

31. The original Latin has been rather freely translated. It reads: "Ad Problemata haec Physica, quae altiuos non spectant quam ad Veri-similitudinem, Propositiones adjunxi Geometricas aliquas, quas Professores Geometriae, & praeterea alii complures Mathematici nostri non recipiunt, sed Calculis suis Arithmeticis videntur sibi confutasse. Ego contrà Arithmeticam illam qua usi sunt, nec confutandis nec confirmandis Pronunciatis Geometricis idoneas esse solus hactenus contendo. Vos ergo nunc appello (qui nondum praejudicio edito mea haec condemnastis Lectores Mathematici) ut dissensionem hanc inter Calculum meum Geometricum, & illorum Arithmeticam (ipsius Mathematicae causa) rationibis vestris componere, & veritati undecunque apparenti succurrere velitis, humillimè oro obsecroque."

supporters, and by 1668, when he was preparing his *Opera Philosophica* for publication at Amsterdam, Hobbes was no longer prepared to submit his geometric work to the judgment of prevailing mathematical opinion.

In the defense of his cube duplication in the *Problemata Physica* against the refutations of the "algebraists" Hobbes returns to the theme of the difference between geometric multiplication (the drawing of lines into lines) and the algebraic or arithmetical multiplication of one number by another. He complains that Wallis's refutation commits an equivocation, since it assigns the quantity *one* indifferently to a line, a surface, and a solid: "In this calculation, if we follow the sound of the words, unity does not change, but (as is customary and necessary to do arithmetic) it is kept the same, namely the line *DV*. . . . But if unity is understood to be changed, as it is first one line, then one plane, and lastly one solid, . . . the calculation can come close to the truth but not attain it" (*PP; OL* 4:382–83). The "small error" that Hobbes claims to find arises, as before, from double-counting of the vertices of a cube: "But when *AS* is converted into numbers, these numbers will be parts of the cube thus made, and the same is counted three times, whence arises the small difference between 1682 and 1681" (*PP; OL* 4:383).

Hobbes's hopes that the *Problemata Physica* and its associated mathematics might receive a favorable hearing from Continental mathematicians were in vain. Huygens (evidently at the request of several members of the Royal Society) responded to the invitation at the end of *Problemata Physica* to submit his thoughts on Hobbesian geometry to Andrew Crooke (Hobbes's bookseller). The result was a letter in which Huygens refuted both the duplication of the cube and the quadrature of the circle. In objecting to the duplication, Huygens observed that Hobbes's construction fails to achieve the desired proportionality. He then dismissed Hobbes's entire solution, together with its defense against the criticisms of Wallis and Rooke with these words:

> But it is extraordinary that he did not notice that his demonstration was flawed, given that it made no mention of what had been assumed in the construction of the problem, namely that AS was taken to be equal to the semidiagonal AI. This assumption, if it were valid, ought to have been a necessary part of the demonstration.
>
> Moreover, he seems to reject without cause the use of arithmetical calculation in the investigation of geometrical construc-

> simply multiplying the number, but rather applying the numbered things, namely 46 lines to 46 lines, I shall show that the calculation is false even on that basis. For in that case, the product of the first multiplication, namely 2,116, will be so many squares of one of the said lines; and therefore even if its side is 46 lines, its square root will still be 46 squares on the said lines. For in every extraction of a numerical square root the numbered thing is the same as in the number from which the root is extracted. For example, the mean proportional (which is the root) between 2 squares and 8 squares is 4 squares, not 4 lines. . . . By neither of his calculations, therefore, has he undermined my duplication of the cube. Nor could it be undermined by the use of numbers even if it were false—unless the squares are divided into an infinite number of parts, so that an infinitely small part (if one may use such a phrase) can be said to be equal both to its square and to its cube. (Hobbes to Sorbière, 19/29 December 1663; *CTH* 2:582–83)

The crux of the matter here is Hobbes's declaration that "in every extraction of a numerical square root the numbered thing is the same as in the number from which the root is extracted." To unravel the doctrine, it is necessary to recall Hobbes's definitions of the terms *point, line,* and *square.* A point is a body whose magnitude is not considered in a demonstration; a line is the trace of a moving point; and a square is a figure generated by drawing a right line through another right line. This means that the side of a square is "inconsiderable" with respect to the area of the square, i.e., there can be no ratio or comparison between them. Hobbes then assumes that the root of anything must be homogeneous with or comparable to the thing whose root it is. Thus, the root of a number is a number, the root of a surface is a surface, etc. This requirement of homogeneity does have some plausibility if we think of roots as *parts* of the quantities of which they are roots; but Hobbes needs some kind of motivation for this doctrine. Further, as attractive as this doctrine may sound when stated abstractly, it amounts to the declaration that arithmetic or algebra cannot be applied to the investigation of geometry. De Sluse was naturally not impressed by such a nonstandard usage of the term *root* and replied that he had calculated in the way that anyone who understands arithmetic, geometry, or algebra would calculate.[34]

34. De Sluse writes, "The only reply I think I should make here is that I calculated in the same way that is normally used by all the arithmeticians who exist, who have

Hobbes's remark that algebra could apply accurately to geometry only "if the squares are divided into an infinite number of parts" suggests that he thought of the size of the algebraic "error" as depending upon the number assigned to a line in a geometric calculation. Recalling his thesis that the error arising from the application of numbers to geometric magnitudes arises from a "double counting" of the vertices of a figure, it appears that Hobbes held that the larger the number assigned to the side of a square, for example, the larger the number of square units it contains, and thus the proportion of squares that overlap in the four vertices of the large square becomes less significant. The problem with such a view is obvious enough: it simply makes no sense. All that is necessary in order to assign numerical values to geometric objects is that they are the sort of thing that can be measured. In fact, nobody ever actually held that the root of a square is its side. Instead, the square root of the quantity that is the *area* of the square (measured in square units) is the number assigned to the *side* of the square (measured in linear units).

The uniform rejection of his mathematical efforts from every corner of the learned world eventually soured Hobbes on the whole of mathematics, at least as it was understood by his contemporaries. We can see a decisive move in this direction in a letter from March of 1664 to Sorbière, where Hobbes declares that he does not "think it worth replying to the critics of my demonstrations (which, after so many explanations, are now being published)." He dismisses all of his critics in language that is clearly that of a man with no interest in reconciling his differences with the tradition:

> I do not want to change, confirm, or argue any more about the demonstration which is in the press. It is correct; and if people burdened with prejudice fail to read it carefully enough, that is their fault, not mine. They are a boastful, backbiting sort of people; when they have built false constructions on other

existed, and (I confidently predict) who will exist in years to come. Nor is there any reason why he should say that I am constrained by reverence or prejudice, since in these sciences nothing can make me submit to authority, and no authority can prevent me from refusing to accept this duplication of the cube of his—even though I value his authority (which is valued by the learned) in literary matters" (de Sluse to Sorbière for Hobbes, 18/28 January 1664; *CTH* 2:614). Hobbes's reaction was to denounce de Sluse as one who "having been warned of his error," refused to embrace the truth (*PRG* 18; *OL* 4:440). Observing this apparently irreconcilable difference in opinion between the two, Sorbière thought he might enlist Fermat as a judge in the contest—a plan de Sluse insisted was not worth undertaking and was in any case prevented by Fermat's death.

> people's principles (which are either false or misunderstood), their minds become filled with vanity and will not admit any new truth. Can a man who believes in the following prodigies really be a suitable judge of my proposition (which is a little more deeply thought out): ten times ten lines are 100 squares; the side of a square and the root of a square number are the same thing; a ratio is a quotient; the same point can be on a line, and outside it, and inside it? (Hobbes to Sorbière, 7/17 March 1664; *CTH* 2:603)

This very hard line against his mathematical opponents is followed by Hobbes's remarkable expression of "a doubt concerning Euclid, book 1, proposition 47" in the very same letter to Sorbière (Hobbes to Sorbière, 7/17 March 1664; *CTH* 2:608–10). This proposition is none other than the Pythagorean theorem, the very result whose demonstration supposedly was the source of Hobbes's infatuation with geometry several decades earlier.

The source of Hobbes's "doubts" is the same as his dismissal of his critics, namely his insistence that the doctrine of roots must be interpreted in accordance with his principles. By this time Hobbes had convinced himself that the entire tradition of geometry—including such classical figures as Euclid and Archimedes—had been misled by thinking that arithmetic or algebra could be applied to geometry.[35] The inevitable result was that Hobbes's program for the reform of mathematics collapsed into self-contradiction and incoherence.

6.2 HOBBES UNREPENTANT

The final stage in the demise of Hobbes's geometrical program is closely connected with the Royal Society and can best be interpreted against the background of his conflict with that institution. In point of fact, some of what we have already seen concerning the duplication of the cube is related to Hobbes's conflicts with the Royal Society. Huygens's refutations of Hobbes's cube duplication, for example, were both solicited by Sir Robert Moray on behalf of the society, with the second one being warmly received at its meeting of 31 December 1662/10 January 1663 (*HOC* 4:295; Birch [1756–57] 1968, 1:167). Hobbes's isolation from the mathematical and scientific community

35. In the letter replying to de Sluse's objections, Hobbes remarks that Archimedes himself "was mistaken in his application of numbers to geometry" (Hobbes to Sorbière, 19/29 December 1663; *CTH* 2:584).

a club bore" (Skinner 1969, 238). Shapin and Schaffer find this explanation superficial and demand ideological reasons for Hobbes's exclusion. They conclude that Hobbes's alleged "dogmatism" was a threat to the form of life exemplified by the society because it represented a rationalistic, absolutist program for the organization of society and the production of knowledge. They insist that his approach to questions of science and politics (which they regard as inseparable) was at odds with the experimentalism and moderate political program of the Royal Society. In consequence, the exclusion of Hobbes was really an attempt to protect the Royal Society's preferred form of politics from an outsider's challenge (Shapin and Schaffer 1985, 139). In other words, complaints about Hobbes's dogmatic and argumentative temperament are really a cover for deeper concerns having to do with his opponents' shared view of how best to maintain a stable social structure.

Noel Malcolm offers a much more nuanced interpretation of Hobbes's exclusion, one that fits together both personal and "ideological" factors. He observes that much of the opposition to Hobbes seems to have been rooted in the contingent accidents of history—Hobbes made enemies for reasons having little to do with "deeper" world-historical forces, and these enemies happened to be leading lights of the British scientific establishment.[37] On the other hand, Hobbes's theology and politics were highly controversial, and it was prudent for those seeking public support for scientific inquiry in the 1660s to distance themselves from one, like Hobbes, who had a reputation as a proponent of dangerous views. Malcolm argues that it is precisely the very significant points of agreement between Hobbes's scientific project and that of the society, combined with his reputation of atheistic materialism, that made it imperative for the society to exclude him for the sake of a proper public image. He writes: "Hobbes was becoming an increasingly disreputable figure, both politically and theologically; and the people who felt that it was most in their interests to blacken his reputation further were the ones who were vulnerable to embarrassing comparisons between his position and their own" (1988, 60). The result, quite naturally, is that there was little sentiment for including Hobbes in the Royal Society.

37. Malcolm writes, "Looking back on Hobbes's disputes with Ward, Wilkins, and Wallis in the 1650s, one is struck at first by the contingency of it all; if only the Universities had not felt politically threatened in 1653–54, one feels, Hobbes would never have become embroiled in these disputes, and would never have suffered the running sore of his mathematical controversy with Wallis—one which did in the end damage his reputation as a scientist" (Malcolm 1988, 57).

the king with his scientific credentials, and one salient reason for such self-promotion is Hobbes's interest in making his way into the society. That Moray would seek Huygens's opinion on the matter, and that Brouncker would be so eager to refute Hobbes's efforts, also indicates that there were members of the society who were interested in making sure that his work was not taken seriously. Charles was evidently unimpressed by Hobbes's campaign, as we can glean from Sorbière's account of an audience with the monarch. He reports that they both agreed that "if he had been a little less dogmatic, he would have been very useful to the Royal Society. For there are few who can see farther into things than he" (Sorbière 1664, 97). Hobbes's "dogmatism" is a theme that recurs throughout the polemics of Wallis and Ward, and in light of Hobbes's stubborn refusal to give ground on the question of the cube duplication, it is clear that Hobbes deserved this much of his reputation.

The role of the cube duplication in Hobbes's battles with the Royal Society is illustrated in an interesting note from the Danish Scholar Ole Borch, who was a correspondent and acquaintance of Sorbière. His journal records a meeting with Sorbière in 1664 in which they discussed a letter in which "Hobbes tries to show that he has found a duplication of the cube; but Sorbière said there is someone else who thinks that he is playing with a paralogism. To which, however, Hobbes has already replied that that man is not a fellow of the Royal Society, and that it is against that society that Hobbes is arming himself."[44] It therefore appears that Hobbes saw his cube duplication as a means both to gain admission to the Royal Society and, by successful defense of his claims, to exact some revenge upon his enemies.

Whatever the full story of Hobbes's relationship with the society, he regarded the "Greshamites" with a mixture of longing for acceptance and resentment at his exclusion. Hobbes's public controversy with the Royal Society opened in 1661 when he attacked the experimental philosophy of Robert Boyle with his *Dialogus Physicus*—a work that, as we have seen, contained a revised version of his earlier cube duplication. This drew two printed responses: Boyle's *Examen of Mr. T. Hobbes, his Dialogus Physicus de Natura Aëris* (Boyle 1662) and Wallis's *Hobbius Heauton-timorumenos; or A Consideration of Mr. Hobbes his Dialogues in An Epistolary Discourse Addressed to the Honourable Robert Boyle, Esq.* (Wallis 1662). The latter is a full-scale

44. This report from *Olai Borrichii itinerarium* is taken from Malcolm's notes to Hobbes's correspondence (*CTH* 2:584, note to letter 161).

plies that π is greater than 3.154, and Huygens had refuted it in his letter to Sir Robert Moray, which was read before the Royal Society in December of 1662. He there challenged Hobbes to "look, then, at my theorem on that subject in my little book *De Circuli Magnitudine Inventa,* where he will indeed find it done without numbers or any numerical calculation whatsoever" (Huygens to Moray for Hobbes, 10/20 December 1662; *CTH* 2:538). Hobbes's response was the twenty-first chapter of *De Principiis et Ratiocinatione Geometrarum,* but before we turn to a consideration of it, it is necessary to have some acquaintance with the basic ideas behind Huygens's work.

Huygens's 1654 treatise *De Circuli Magnitudine Inventa* was a decisive advance in the attempt to determine the value of π, or, equivalently, to determine the area of the circle exactly.[46] Archimedes had given a method of calculating π to within any desired degree of accuracy, but the method was as cumbersome as it was rigorous. The Archimedean procedure (which I outlined briefly in chapter 1) is based on the principle that the perimeter of a regular polygon inscribed in a circle is less than the circumference of the circle, while the perimeter of a similar polygon circumscribed about the circle exceeds the circumference. By increasing the number of sides in the inscribed and circumscribed regular polygons, these upper and lower bounds can be systematically improved and made to converge upon the value of π. Thus, using Archimedean methods, we can construct a sequence of inscribed regular polygons with perimeters $p_1, p_2, \ldots, p_n \ldots$ and a sequence of circumscribed polygons with perimeters $P_1, P_2, \ldots, P_n \ldots$ such that (assigning the diameter of the circle a unit value) $p_1 < p_2 < \ldots < p_n \ldots < \pi < \ldots P_n < \ldots P_2 < P_1$. Archimedes himself inscribed and circumscribed hexagons about a circle, then successively doubled their sides to produce two ninety-six-sided figures, which yields the inequality $3.14084 < \pi < 3.142858$.

Although the Archimedean procedure can give arbitrarily precise approximations to π, it is quite cumbersome and involves horrendously difficult calculations because the approximating sequences converge very slowly to the desired value. In a truly remarkable exhibition of perseverance through sheer tedium, the Dutch mathematician Ludolph van Ceulen calculated π to thirty-five digits, using Archimedes' procedure. Willebrord Snell, another Dutchman, sought a faster means to determine the value of π. In his *Cyclometricus* Snell used, although he

46. See Hoffman 1966 for a detailed study of this work, its background, and its influence.

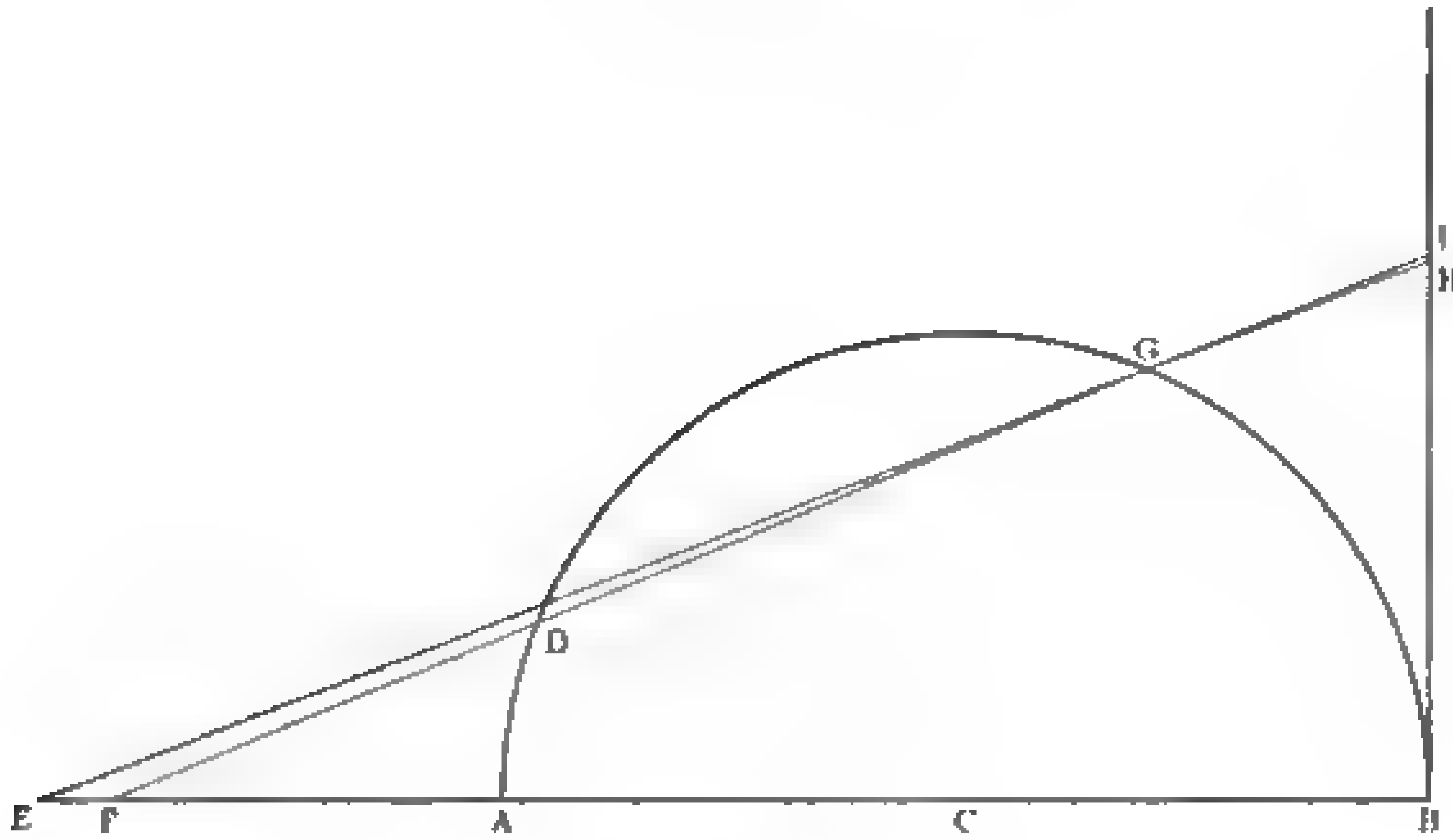

Figure 6.2

did not rigorously prove, geometric principles that enabled the value to be calculated more efficiently.[47] Huygens found rigorous geometric proofs of Snell's principal results, which can be stated as follows. If the line EA is chosen so that $EA = AB/2$ (as in figure 6.2), then $\widehat{BG} >$ the tangent BH; similarly if DF is chosen so that $FD = AB/2$ (again as in figure 6.2), then $\widehat{BG} <$ tangent BI. The effect of instituting such bounds is to accelerate the convergence of the sequences $\{p_n\}$ and $\{P_n\}$ toward the common limit of π and thus to give much more precise approximations toward the sought value.

In addition to giving a rigorous derivation of Snell's methods, Huygens also established a number of other results concerning the area of a segment of the circle and that of inscribed or circumscribed triangles. These can be stated with reference to a segment ABC less than a semicircle (whose area we can designate as a), triangle ADC (formed by the base and tangent lines AD, CD) and triangle ABC (the largest triangle that can be inscribed in the segment), as in figure 6. 3. Huygens showed that 2/3 triangle $ADC > a >$ 4/3 triangle ABC. With the aid of these and other theorems, Huygens showed that the Archimedean approximation procedure can be accelerated. In particular, under the appropriate choice of inscribed and circumscribed regular polygons, if $p_n < \pi < P_n$ we can improve the approximation by exploiting the fact

47. These two results appear as propositions 28 and 29 in Snell 1621, 42–45. They are demonstrated in theorem 12 of Huygens's *De Circuli Magnitudine Inventa* (*HOC* 12:156–59). Cantor ([1900–1901] 1965, 2:704–17) contains an overview of this and related material.

from the summary he gives of the principal result. In Hobbes's recapitulation, Huygens is supposed to have argued as follows:

> Let a circle segment *ABC* [figure 6.4] less than a semicircle be described, and let it be divided in two equal parts by the perpendicular *FB*. And further having bisected the arcs *AB*, *BC* at *D* and *E*, and having drawn their chords *CE*, *EB*, *BD*, and *DA*, and the tangents *CH*, *BH*, *BI*, *IA*, and *CK*, *KE*, *EL*, *LB*, and *BM*, *MD*, *DO*, *OA*; and letting these be successively bisected as far as can be understood *[quantum intelligi potest]*, he demonstrates (and, as far as I can see, correctly) that the circle segments *CEC*, *EBE*, *BDB*, *DAD* are less than the triangles *CKE*, *ELB*, *BMD*, *DOA*. (*PRG* 21; *OL* 4:451)

This corresponds to no particular theorem in Huygens's treatise, although it may be Hobbes's attempt to reproduce the exhaustion proofs that establish some of the ratios between circular segments and inscribed and circumscribed triangles.[50] Hobbes's defense of his results against Huygens's reasoning is truly remarkable. He insists:

> But when he likewise infers that, if by perpetual bisection an infinite number of segments are made, these also taken together will be less than all of the triangles, corresponding to the segments, taken together, he infers badly, unless the right line *IBH* is outside the circle, so that the point *B* is not common to both the right and curved lines but between them. For if *B* is common to both lines, then all the infinite number of tangents constitute the arc *ABC*. But if *B* is outside the circle, however much it may touch the circle, the chords *AB*, *BC* will not cut the circle in the same point in which they cut the right line *FB*, but will be short of it on both sides. But Euclid teaches (*Elements* 3, prop. 2) that the right line *CB* is entirely inside the circle, and (*Elements* 3, prop. 16) that the tangent is entirely outside the circle. And so no right line except *FB* can go through the arc and tangent at the same point *B*, namely at the point that is called that of *contact*, unless both lines are attributed some latitude. (*PRG* 21; *OL* 4:452)

50. Particularly that in theorems 3 and 4 of *De Circuli Magnitudine Inventa*. Hobbes himself admits the likely inadequacy of his presentation: "Consult his book itself, my copy of which is not with me as I write; nor if it were here, would his demonstrations easily be transcribed" (*PRG* 21; *OL* 4:452).

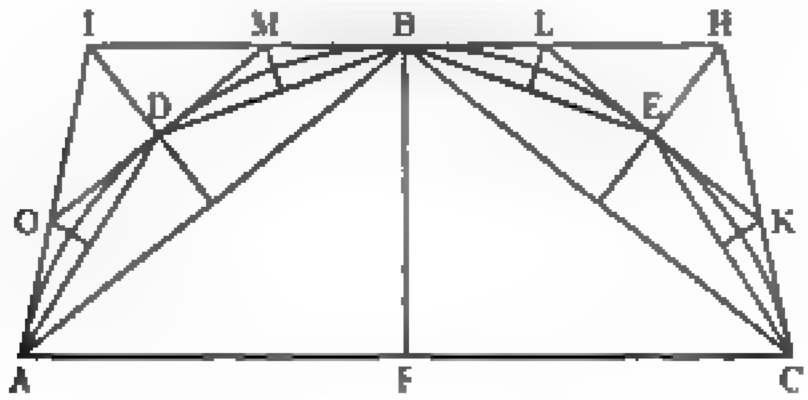

Figure 6.4

Hobbes then declares that from the use of the false conception of points as "nothing,"

> after many demonstrations, [Huygens] inferred that "the difference between a third part of the arc of a quadrant and the chord, is to the difference between the chord of the same third part of the arc and its sine . . . in a ratio greater than 4 to 3," which is insufficient to confirm what he wanted to refute. But when he inferred that the right line composed of a radius and the tangent of thirty degrees is greater than the arc of the circle, he fell into error, because he thought that the radius of the circle is not less than that which is drawn from the center to a tangent, which is outside the circle. (*PRG* 21; *OL* 4:452)

This response can only be interpreted as an act of desperation. In effect, Hobbes seeks to protect the validity of his quadrature by attributing any difference between his and Huygens's results to the latter's neglect of point extensions. In this context it is important to recall that Hobbes had voiced no qualms about Huygens's central theorems in 1656, when Mylon had used them as the basis for objections to the treatment of parabolic arcs in chapter 18 of *De Corpore*. In the intervening ten years Hobbes had obviously changed his estimation of such methods.

The price to be paid for such an insistence on the correctness of his quadratures is Hobbes's ultimate rejection of such geometric basics as the Pythagorean theorem. We have already seen how, in a 1664 letter to Sorbière, Hobbes expressed "doubts" about the Pythagorean theorem. By 1666, he was more emphatic. He remarks in Chapter 23 of *De Principiis et Ratiocinatione Geometrarum* that "if these things of mine are correctly demonstrated, there are some further things that you should consider. First, the greater part of the propositions that depend on proposition 47 of book 1 of Euclid (and there are many)

are not yet demonstrated. Second, the tables of sines, tangents, and secants are wholly false" (*PRG* 23; *OL* 4:462–63).

The publication of *De Principiis et Ratiocinatione Geometrarum* did nothing to enhance Hobbes's reputation. In a brief and dismissive review published in the *Philosophical Transactions,* Wallis highlighted the essentially hopeless condition of Hobbes's geometric program. He recounts the difficulties facing the alleged quadratures:

> For finding himself reduced to these inconveniences; 1. that his *Geometrical Constructions,* would not consist with *Arithmetical calculations,* nor with what *Archimedes* and others have long since demonstrated: 2. That the *Arch* of a Circle must be allowed to be sometimes *Shorter* than its *Chord,* and sometimes *longer* than its *Tangent:* 3. That the Same Straight Line must be allowed, at one place onely to *Touch,* and at another place to *Cut* the same Circle: (with others of a like nature;) He findes it necessary, that these things may not seem Absurd, to allow his *Lines* some *Breadth,* (that so, as he speaks, *While a Straight Line with its Out-side doth at one place Touch the Circle, it may with its In-side at another place cut it,* &c) But I should sooner take this to be a *Confutation* of his *Quadratures,* than a *Demonstration of the Breadth of a* (Mathematical) *Line.* (Wallis 1666, 290–91)

It is hardly necessary to stress that Hobbes was not taken seriously by the mathematical world after the publication of *De Principiis et Ratiocinatione Geometrarum.* Where he had been both able and willing to gain a hearing for his mathematical work before the mid-1660s, he no longer corresponded with mathematicians in England or on the Continent, and his mathematical works were generally ignored.

6.3 ENDGAME: HOBBES'S LAST PUBLICATIONS ON MATHEMATICS

From 1668 until his death in 1679, Hobbes continued to publish an astounding amount of mathematical material, but he had set himself so firmly against prevailing mathematical opinion that he was entirely impervious to any criticism. These publications include two compendia of Hobbesian mathematical writing, *Rosetum Geometricum* and *Principia et Problemata,* as well as four papers addressed to the Royal Society, the *Lux Mathematica,* two different editions of a 1669 pamphlet claiming to square the circle, and a mathematical appendix to the *Decameron Physiologicum.* There is nothing to be gained by a detailed

examination of all these works, but some key points can be clarified by looking into some of them briefly.

In 1668 Johannes Blaeu published a collection of Hobbes's works in Amsterdam under the title *Opera Philosophica.* This gave Hobbes the opportunity to revise his earlier publications, and it was also the occasion for the Latin translation of *Leviathan.* The 1668 revisions to the mathematical sections of *De Corpore*—especially the circle quadrature in chapter 20—are especially relevant because they document Hobbes's retreat from the principles and practices of European mathematics. In the original (1655) version of chapter 20 (and in the English version from 1656) Hobbes began his attempted quadrature with a brief resume of earlier approaches to the problem, whose validity he clearly accepted. Hobbes there informs us that "[a]mongst those Ancient Writers whose Works are come to our hands, *Archimedes* was the first that brought the Length of the Perimeter of a Circle within the limits of Numbers very little differing from the truth; demonstrating the same to be less than three Diameters and a seventh part, but greater than three Diameters and ten seventy one parts of the Diameter" (*DCo* 3.20.1; *EW* 1:287). Such a positive evaluation of previous efforts is not confined to Archimedes, for Hobbes mentions "*Ludovicus Van Cullen & Willebrordus Snellius*" as having "come yet neerer to the truth; and pronounced from true Principles, that the Arch of a Quadrant (putting, as before 10000000 for Radius) differs not one whole Unity from the number 15707963" (*DCo* 3.20.1; *EW* 1:287–88).

The 1668 version of chapter 20 of *De Corpore* opens, not with a review of previous approximations to π, but with an argument designed to show that the sine of 30 degrees (taken to seven decimal places) is .5811704. This value differs significantly from the accepted value of .5773503, and Hobbes takes it upon himself to explain why his result should differ from the commonly accepted one. His opponents are in error, Hobbes explains, and their error has its origin in the view that the multiplication of numbers is the same thing as the multiplication of lines:

> The cause of the error is not that the numbers were falsely counted up, but that a number multiplied into a line never produces the same thing as a line drawn into just as many lines. And thus the root of a square number is never the side of a square figure, whatever the number of its parts may be. If you divide a right line, for example, into three equal parts, and you multiply the same line by the number three, there will not be produced a

> square but nine lines. Or if you divide this line into 10,000,000 parts, the product of these parts and [the number] 10,000,000 will be so many lines, not so many squares on these parts. For in order to make a rectangle from a line and a number it is necessary that the number be infinite. (*DCo* 3.20.1; *OL* 1:243–44)

This by now familiar doctrine is underwritten by Hobbes's insistence that "in every extraction of roots from a number, the root will be an aliquot part of its square"; similarly, the traditional method of computing sines is faulted for its use of arithmetical calculation in a geometric setting. Archimedes' result, which Hobbes had earlier accepted, is now rejected because "even Archimedes' measure of the circle, which depends from the beginning upon this calculation, is less than the true value; nor can it succeed in refuting a geometric calculation in which the perimeter of the circle happens to be found greater than that found by him" (*DCo* 3.20.1; *OL* 1:245).

Hobbes's 1669 pamphlet *Quadratura Circuli, Cubatio Sphaerae, Duplicatio Cubi, Breviter demonstrata* deserves a brief mention in this context because it contains a new assault on the quadrature of the circle that (perhaps not surprisingly) yields a result inconsistent with the quadratures Hobbes had defended just three years earlier.[51] Take the circle *BCDE* (figure 6.5) with center *A*, divided into four equal sectors by the diameters *BD* and *CE*. Construct the square *GHIF* to circumscribe the circle and touch it at the points *B, C, D*, and *E*. Draw the diagonals *GI* and *FH*, cutting the circle at points *K, L, M, N*. Bisect the line *GC* at *O*, and draw the line *AO*, cutting the circle at *P*. Through *P* draw a line parallel to *GC*, to intersect *AG* in *Q* and *AH* in *R*. With *QR* as a side, construct the square *QRST*. Hobbes asserts that the *QRST* is equal in area to the circle *BCDE*. The argument for this remarkable result is not sufficiently interesting to detain us; the salient fact is that Hobbes derives the conclusion by introducing a key assumption in the form of a disjunction, viz.: "Either there can be no right triangle in *ACG* with vertex *A* equal to the sector *ACL*, or the areas *PQL, CYP* are equal" (Hobbes 1669a, 2). Because the first disjunct is obviously false (i.e., there must be *some* rectilinear triangle similar to *ACG* with vertex *A* and equal in area to *ACL*), Hobbes concludes that the areas *PQL, CYP* are equal, and then quickly concludes that the square *QRST* is equal to the circle *BCDE*.

However, Hobbes conveniently overlooks the possibility that both

51. The full argument for this quadrature is contained in section 6 of the appendix. My treatment of it here is consequently quite brief.

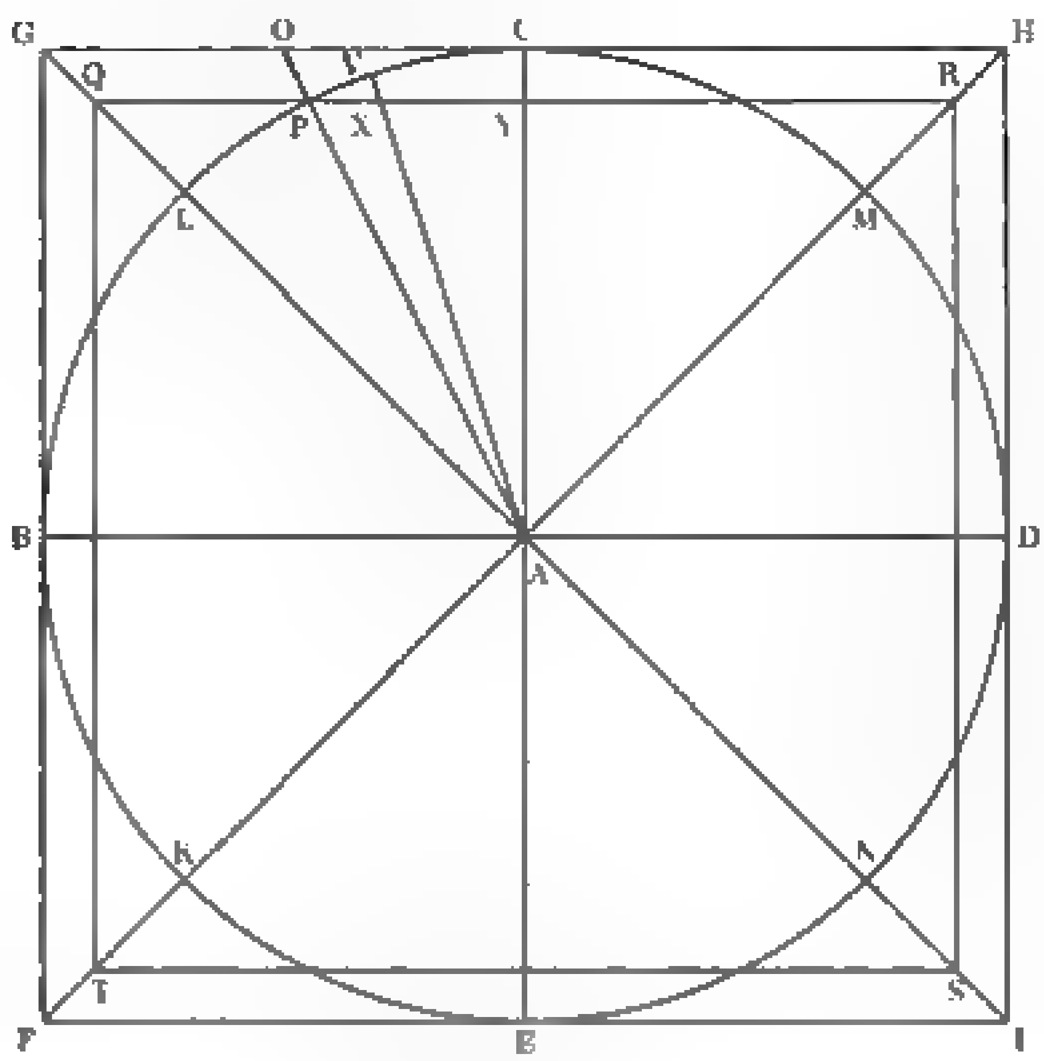

Figure 6.5

disjuncts of his assumption are false, and indeed, his argument does little more than beg the question at issue. Furthermore, his argument has the unfortunate consequence of setting the value of π at 3.2—a result that Wallis happily demonstrated in his rebuttal *Thomae Hobbes Quadratura Circuli, Cubatio Sphaerae, Duplicatio Cubi Confutata* (Wallis 1669a, 3). Hobbes was evidently prepared for such a reaction and in the second edition of the *Quadratura* he replies to Wallis's objection. This work was dedicated to Prince Cosimo de'Medici, and in a letter to the prince Hobbes explains that "I did not want to abuse the patronage of such a great prince by invoking it on behalf of false or uncertain doctrine. So, in order that the earlier copies of this pamphlet should be subjected to the most minute examination, I submitted them not to friends but, more cautiously, to torturers—putting them, as it were, in a purifying fire" (Hobbes to Cosimo de'Medici, 6/16 August 1669; *CTH* 2:711). Hobbes's reply to the central objection is to complain that when Wallis

> proves that the triangle *AYQ* and the sector *ACL* are not equal, he assumes as demonstrated by Archimedes that the perimeter of the circle to the diameter is in a ratio less than that of 22 to 7. But this is neither demonstrated by Archimedes, nor can it be demonstrated by his method. For it proceeds from the extraction of roots, and assumes that the root of a number of squares con-

> tained in a greater square is the side of the whole square, which is manifestly false. For the root of a number of squares is some other number of squares, just as *[non aliter quam]* the root of 100 stones is 10 stones. (Hobbes, 1669b, 2; *OL* 4:508)

The utter collapse of Hobbes's geometric ambitions was complete by 1670. He continued to publish mathematical tracts and defenses of his various alleged results, but seems to have had no readership. Hobbes undertook the hopeless task of gaining a hearing for his views from the Royal Society, to whom he addressed a collection of papers and his *Lux Mathematica,* but the society's only response was Wallis's scathing reviews of his work in the *Philosophical Transactions.* He also printed three separate papers addressed to the Royal Society, specifically taking issue with Wallis's mathematics and defending his own thesis that algebraic calculation can never challenge a geometric result. These also gained no hearing for his views other than giving Wallis a further opportunity to disparage his opponent in the pages of the *Philosophical Transactions.* Some ten years before he actually died, Hobbes was a mathematical casualty and his program for geometry did not survive the death of its founder and lone adherent.

CHAPTER SEVEN

The Religion, Rhetoric, and Politics of Mr. Hobbes and Dr. Wallis

> I can, after all, write the remaining parts of my Philosophy, and at the same time write letters to you, and moreover give a thrashing in passing to some worthless fellow who treats me impertinently. My quarrel with him is not like the quarrel between Gassendi and Morin or Descartes. I was dealing at the same time with all the ecclesiastics of England, on whose behalf Wallis wrote against me. Otherwise I would not consider him the least bit worthy of a reply, whatever is thought of his books by certain rather famous geometers—who are also rather ignorant of the art they teach, as will perhaps become more obvious shortly.
>
> —Hobbes to Sorbière, 29 December/8 January 1657

The preceding six chapters have been primarily concerned with the mathematical aspects of the war between Hobbes and Wallis. There is good reason for this, since their dispute was first and foremost over mathematical matters. Nevertheless, it would be misleading to characterize the exchange of polemics as concerned exclusively with mathematics. As I showed in chapter 2, nonmathematical issues involving religion and politics were crucial to the early stages of the controversy. In fact, it is unlikely that Wallis or Ward would have bothered refuting Hobbes's mathematical claims so publicly and at such length had they had not seen him as a danger to the universities and religion. Indeed, we saw that Hobbes's decision to publish his circle quadratures in *De Corpore* was in all likelihood prompted by Ward's baiting in the *Vindicae Academiarum*. But political and religious issues were not confined to the opening stages of the controversy. Extramathematical material appears alongside debates over purely mathematical issues in many of the exchanges between the two combatants. Thus, to take one

example, the question of how best to interpret the doctrine of proportions is taken up in the first part of Hobbes's Στίγμαι, to be followed several pages later by his attack upon Wallis's conception of Christian ministry, which itself is followed by a seemingly unrelated discussion of subtleties of Latin grammar. My task in this chapter is to cover those aspects of the Hobbes-Wallis controversy that do not concern mathematics but are nevertheless important components of the war between these two.

This chapter is organized into three sections that cover the most salient extramathematical aspects of the dispute. The first covers the thorny issues of religion and church government. The second is a summary of the various charges of grammatical and rhetorical infelicity traded by the disputants. The third deals with political questions, and particularly the issue of loyalty to the restored monarchy. I make no claim to comprehensive coverage of these aspects of the dispute, but I offer an overview complete enough to acquaint the reader with the relevant contested issues.

7.1 HOBBES AND WALLIS ON RELIGION

Religious conflicts were practically omnipresent in seventeenth-century Europe, and they provide something like a fixed frame of reference for the intellectual historian. One way of locating a philosophical figure from this period on the conceptual map is to investigate his stand on the contested religious questions of the day. Indeed, it is arguably the case that almost any significant seventeenth-century controversy contains a religious element. Hobbes and Wallis disagreed about nearly everything, so it is no surprise that their views should diverge on questions of religion. Hobbes's religious opinions stood far outside the mainstream of seventeenth-century thought, although the extent of his heterodoxy remains a matter of debate and will be of concern to us later. Wallis, in contrast, fits fairly neatly into the reformed tradition, and particularly in the Presbyterian strain of English Puritanism. Here again, as in many mathematical aspects of their dispute, their conflict pits the "outsider" Hobbes against the traditionalist Wallis. This is not to say that the two men's doctrinal differences made them incapable of understanding each other when issues of religion arose, but it does reinforce the idea that Hobbes's philosophical, mathematical, and religious doctrines placed him in opposition to received views.

Two important issues are at the heart of the religious dimension of

the Hobbes-Wallis dispute. The first is that of church government, i.e., how the church ought to be organized and the extent of the powers granted to the church hierarchy to enact and enforce laws binding on the populace. The second is the charge (raised by others besides Wallis) that Hobbes is an atheist, notwithstanding his numerous professions of religious belief. These two topics merit separate treatment and will be covered in two subsections.

7.1.1 *The "Scottish Church-Politics" of Dr. Wallis and the Limits of Sovereign Power*

The issue of church government was of paramount importance in seventeenth-century England. The Civil War was, among other things, fought over questions of how the church should be organized and who should hold the power of determining its doctrine, liturgy, and forms of worship. I cannot give an exhaustive account of the issues in play in this disputed area, but the main lines of the conflict can be discerned quite readily. There were three principal forms of Christianity in mid-seventeenth-century England: Catholicism, Anglicanism, and Puritanism. These obviously differ in many respects, but one of the most important points of difference is on the question of church government, and it is against the background of such differences that Hobbes and Wallis traded polemics on the question of church authority. Although officially suppressed in England after the reign of Mary (1553–58), Catholicism survived surreptitiously into the seventeenth century, and with it the doctrine of the pope's supreme authority in matters of religion. Catholicism also had a distinctive episcopal structure in those countries where it was officially established. Catholic bishops and canon courts claimed authority in church matters and constituted essentially a parallel governmental structure alongside that of the civil sovereign.[1]

1. Hobbes is an excellent source to outline this division between civil and religious powers under Catholicism. In *Behemoth*, he succinctly catalogs the claims made by the pope against the rights of civil sovereigns. These include: "[f]irst, an exemption of all priests, friars, and monks, in criminal causes, from the cognizance of civil judges. Secondly, collation of benefices on whom he pleased, native or stranger, and exaction of tenths, first fruits, and other payments. Thirdly, appeals to Rome in all causes where the Church could pretend to be concerned. Fourthly, to be the supreme judge concerning lawfulness of marriage, that is concerning the hereditary succession of Kings, and to have the cognizance of all causes concerning adultery and fornication. . . . Fifthly, a power of absolving subjects of their duties, and of their oaths of fidelity to their lawful sovereigns, when the Pope should think it fit for the extirpation of heresy." The result is that "[t]his power of absolving subjects of their obedience, as also that other of being

Anglicanism retained the episcopacy but placed this structure under the control of the monarch, who was the supreme authority in civil and religious questions. The 1534 Act of Supremacy established Henry VIII as the head of the English church and seized papal lands for the crown. After a period of reestablished Catholicism under Mary, the Elizabethan Settlement of 1559 issued in Acts of Uniformity and Supremacy that made the monarch the supreme governor of the Anglican Church and imposed uniformity of observance throughout the nation. The result was a national church decidedly Protestant in its theology, with an episcopal structure subordinated to the crown but with ceremonies and liturgy that retained many Catholic elements.

By the seventeenth century Anglicanism was under attack from Puritans who regarded it as insufficiently reformed or fatally tainted by "popish" elements in doctrine, liturgy, governmental structure, and ceremony. Resentment against the episcopacy is a common thread in Puritan writings of the period, and calls for the abolition of the episcopal structure were a commonplace among the reformers of the 1630s. There was considerably less uniformity among Puritans about what form of church polity should replace episcopacy, because the term *Puritan* means little more than "non-Anglican Protestant" and is therefore compatible with a variety of models of church government. For my purposes the principal division in Puritanism is that between Presbyterianism and Independency. The differences between these two models of church government are sufficiently important to be set out briefly, and the best way to do this is to see their interaction in the conflict between the monarchy and Parliament that turned into the English Civil War.

Presbyterianism in England took its lead from the Church of Scotland, which had been reformed along strict Calvinist lines in the sixteenth century. The preferred form of church government among Calvinists was a ruling body of ministers and elders—the presbytery—who answered not to the civil authority, but directly to God. The Presbyterian Church claimed authority over the populace as a whole, and it assumed the power to exact tithes and enforce discipline as it saw fit, without reference to the claims of the civil sovereign, whom Presby-

judge of manners and doctrine, is as absolute a sovereignty as is possible to be; and consequently there must be two kingdoms in one and the same nation, and no man be able to know which of his masters he must obey" (*Behemoth* 1; *EW* 6:172–73). A lucid study of church-state issues in the context of Hobbes's political theory is Sommerville 1992, chap. 5.

terian doctrine takes to be simply another parishioner. In its official doctrine Presbyterianism also has no place for episcopacy, but the reform of Scotland was not so thoroughgoing as to effect the absolute abolition of bishops. Indeed, James VI of Scotland (the future James I of England) managed through a series of measures to retain some portion of episcopal structure in the church and thereby keep it from the most rigidly Presbyterian model (Mullan 1986). No monarch with aspirations to supreme authority could welcome the claims of Presbyterianism, and when James I ascended to the English throne in 1603 he took steps to dilute the power of the Presbyterians and bring Scotland into still closer conformity with the Anglican model. The result was naturally a contentious relationship between the crown and the Scots church, which only intensified with the ascent of Charles I to the throne in 1625. Charles supported a variety of policies that aimed at increasing the power of the crown over the church while seeking a rapprochement with the church of Rome. William Laud, whom Charles made archbishop of Canterbury in 1633, was closely identified with a campaign against Puritan influences and the reintroduction of "Romish" practices in Anglican liturgy and ceremony. With the attempt to impose the English Book of Common Prayer in Scotland in 1638, Scottish Presbyterians rose in open rebellion.

Needing money to prosecute the first of two Bishops' Wars against the recalcitrant Scots, Charles I (who had ruled without a Parliament since 1629) called Parliament into session in the spring of 1640, but adjourned it almost immediately when it began to list grievances against his "personal" (i.e., nonparliamentary) rule. Renewed fighting with the Scots in the second Bishops' War led to the occupation of northern England by Scottish forces, and Charles was forced to call Parliament into session in November of 1640. This Long Parliament would remain in session throughout the Civil War in the 1640s until it was purged by the army in 1648, leaving the radical Rump Parliament, which presided over the execution of Charles I and the abolition of the monarchy. One of the Long Parliament's more important actions was the presentation of a Grand Remonstrance to Charles in December of 1641 that listed a comprehensive catalog of grievances against the crown. On the matter of church government, the members of Parliament insisted that they had no intention "to let loose the golden reins of discipline and government in the Church," because "we hold it requisite that there should be throughout the whole realm a conformity to that order which the laws enjoin according to the Word of God"

(Prall 1968, 72). Nevertheless, the authors of the Grand Remonstrance objected to the episcopalian model as implemented by Laud, and demanded that

> the better to effect the intended reformation, we desire that there may be a general synod of the most grave, pious, learned and judicious divines of this island; assisted with some from foreign parts, professing the same religion with us, who may consider of all things necessary for the peace and good government of the Church, and represent the results of their consultations unto the Parliament, to be there allowed of and confirmed, and receive the stamp of authority, thereby to find passage and obedience throughout the kingdom. (Prall 1968, 72)

This demand for a general synod eventually resulted in the creation of the Westminster Assembly of Divines, which was first convened on 1 July 1643. Charles had opposed the institution of the assembly as an obvious imposition upon his right, as head of the Church of England, to decide matters of church governance. In the course of events, however, Charles's opinion had become largely irrelevant. After the outbreak of general hostilities between Parliamentary and Royalist forces in September of 1642, there was a state of open war between the two sides and the Long Parliament no longer answered to the king.

The Westminster Assembly of Divines was charged by Parliament with advising it on how to institute such a form of church government "as may be most agreeable to God's holy word, and most apt to procure and preserve the peace of the Church at home, and nearer agreement with the Church of Scotland and other Reformed Churches abroad" (Mitchell and Struthers 1874, ix). By the late summer of 1643 the Parliamentary cause was faring badly. Seeking assistance from Scotland, Parliament agreed to a "Solemn League and Covenant" with the Scots, the object of which was "the preservation of the reformed religion in the Church of Scotland, in doctrine, worship, discipline, and government, against our common enemies; the reformation of religion in the kingdoms of England and Ireland, in doctrine, worship, discipline and government, according to the Word of God, and the example of the best reformed Churches" (Prall 1968, 104). One of the key elements of the Solemn League and Covenant was its commitment to "the extirpation of Popery [and] prelacy (that is, Church government by Archbishops, Bishops, their Chancellors and Commissaries, Deans, Deans and Chapters, Archdeacons, and other ecclesiastical officers depending on that hierarchy)" (Prall 1968, 105). The Solemn

League and Covenant was generally understood to oblige Parliament to introduce a Presbyterian form of church governance, but the deliberations of the assembly showed that there was hardly unanimity of opinion on this point.

The label *Independent* was first used to describe those members of the assembly who resisted the institution of Presbyterianism. The clerical leadership of the Independents has been described as "a small group of members of the Westminster Assembly fighting a rearguard action against the apparently inexorable implementation of Presbyterianism between 1643 and 1647" (Mullett 1994, 196). Although Independency had a small following in the early stages of the Civil War, the Independents and their allies ultimately held positions of great power in the army and under the Protectorate. No less a figure than Cromwell was closely identified with the Independent faction, and by the time he assumed the title of Lord Protector in 1653 the Independents were the dominant party while the Presbyterians had assumed the uncomfortable role of an embattled minority. The fundamental point of difference between Independency and Presbyterianism is the autonomy of the congregation. The Independents generally held that the freely gathered congregation was the only true model of the New Testament Church and they resisted attempts to subordinate the congregation to an ecclesiastical hierarchy. Presbyterians regarded such autonomy as intolerable laxity no better than the outright abolition of church government. As ever-more radical sects proliferated in the late 1640s and 1650s, the Presbyterians accused the Independents of fomenting schism and offering no means of controlling the dissolution of religious order.[2]

Wallis was appointed to the Westminster Assembly in 1644, serving as a secretary until the final plenary session in February of 1649. He reports in his autobiography that "I was one of the Secretaries to the *Assembly of Divines* at *Westminster*. Not from the first sitting of that *Assembly;* but some time after, and thenceforth during their Sitting" (Wallis 1970, 31). His sympathies were strongly with the Presbyter-

2. For example, in attacking Independency as a form of schism, Daniel Cawdrey complained that "Toleration (which is our *present condition*) hath done much more towards the *rooting of Religion*, out of the hearts of many men in 7 yeares, than the enforcing of *uniformity* did in 70 yeares." He concluded that the toleration sought by John Owen and the Independents was the source of complete religious disorder: "Let experience speake; If since the men of *his* [i.e., Owen's] *way* have gotten a *Toleration* for themselves, they have not opened a *doore* for all *errours*, *heresies*, and horrid blasphemies, or *profanenesse*" (Cawdrey 1657, 14, 16).

theory regards the ministerial office as part of a system whose authority is guaranteed by the word of God and is not restricted to the members of a single congregation. A proper Presbyterian minister is not simply someone responsible for a specific congregation, but one who has been granted his authority by Christ's universal church and can, at least in principle, authoritatively exercise his ministerial function anywhere.

In arguing for his negative response to the proposed question Wallis first explains that by the term "ministers of the Gospel" he understands "those of whom the preaching of the Gospel is demanded, ex officio, by Christ" (*Mens Sobria* 136). Although the primitive church of the New Testament included apostles and prophets among its ministers, Wallis assumes that in the contemporary context the class of ministers would include only "pastors and doctors" of the church. The powers of interest here are those "which belong to the Gospel minister, by virtue of the office he exercises, in so far as they uphold his public function and distinguish him from a private Christian" (*Mens Sobria* 136). Such powers include "the preaching of the Gospel, the administration of sacraments, the exercise of ecclesiastical censures, the ordination of ministers, and such things of this sort" (*Mens Sobria* 136–37). Wallis remarks that some (i.e., the Independents) conceive of the relationship between the minister and congregation as closely analogous to that between a shepherd and his flock—just as a shepherd is a shepherd only in relation to a specific flock and cannot properly exercise power over the flock of another, so the ministerial powers must be restricted to a specific congregation. He objects that, on the contrary, "the relation of a minister of the Gospel to a particular church is neither adequate nor his primary relation, which is rather that which he bears to Christ, whose minister he is" (*Mens Sobria* 140). As is customary in such disputations, Wallis builds his case on an analysis and interpretation of scriptural passages; he reads references to "the church" or to "Christ's flock" as referring to a universal church that cannot be identified with a specific congregation. Even in those cases where relevant biblical passages clearly seem to intend a specific congregation, Wallis contends that such references

> are made synecdochically, that is by the application of the name of a whole to some of its parts. Just as Christ himself is said to have preached unto the world, although he never, as far as we know, preached outside of the land of the Israelites *[terra Israelitica]*. . . . And so, if some particular church is on occasion syn-

> ecdochically called Christ's family, or the flock of Christ, or even the body of Christ, nevertheless when several are spoken of they are not called the flocks of Christ, or his families, or his bodies, or even his brides. Instead, when these terms are used for one or more or even all, they are understood to be used only to indicate one family, one body, one flock, one bride, and one edifice. (*Mens Sobria* 142–43)

The consequence of this interpretation is that "those titles by which ministers [of the Gospel] are everywhere distinguished do not signal a relationship to this particular church, but instead either to Christ himself, or also to the universal Church, or to the absolute authority of preaching the Gospel" (*Mens Sobria* 146). Wallis concludes his account of the ministry with the declaration that "it is manifest that the ministry of the Gospel is not only of one particular church, but rather the ministry of the whole universal Church; or rather, speaking absolutely, the minister of Christ is not ordained for the good of only one particular church, but for the good of the whole universal Church" (*Mens Sobria* 152).

Such a forthright attack on the fundamental tenets of Independency found no favor with Owen, and Oxford's vice-chancellor subsequently became embroiled in controversy with the Savilian professor of geometry. Daniel Cawdrey reports that "when the learned DOCTOR WALLIS, had brought to him as *Vicechancellor,* that Question to be defended *negativè* in the *vespers* of the publick Act at *Oxford*, 1654. '*An potestas Ministri evangelici, ad unius tantum Ecclesiae particularis membra extendatur,*' this Reverend Doctor [Owen] said thereupon that *Doctor Wallis* had brought him a challenge, adding, that if he did *dispute* upon that Question, he must dispute *ex animo*" (Cawdrey 1657, 129). It is interesting to observe that a year later (1655) Wallis dedicated his *Elenchus* to Owen, and it is natural to see this as an attempt to improve his standing with the vice-chancellor by making common cause against the notorious Hobbes.[4] Nevertheless, Wallis aroused the ire of Owen and his allies when he subsequently published his critique of Independency as part of the *Mens Sobria* in 1656, an action that Owen took to be a direct challenge to his own authority and to Independency in general.

4. Hobbes noticed this dedication and took the chance to denounce Wallis for attempting to "curry favor" with Cromwell's ministers, "as you did by Dedicating a Book to his Vice-Chancellor, *Owen*" (*MHC*; *EW* 4:416).

Henry Stubbe wrote to Hobbes in 1656 and mentioned the ongoing controversy between Wallis and Owen. According to his report,

> [t]here is now a matter of greater diuertisement happened; wch is this: D:r Wallis who is now putting out a most childish answer, (as they say) of 16 sheets in a small letter, to yor *[Six Lessons]*, hath put out some theses agt a branch of independency: D:r Owen hath wrote a booke of late in behalfe of Independency, & our du Moulin another, & it is intended that ye Presbyterians shall be mated here in Oxon: Wallis's theses are triuiall, & so D:r Owen hath desired mee to fall upon him; & to seize him, as I thinke fit, because hee did under hand abuse dr Owen lately. D:r Wallis hath mistooke ye question, & so I haue order to tell him of it, & to reflect upon ye synod at westminster, whereof he was Scribe. this will take mee up for some weekes, for I intend to spend as good Latine as I can upon him, & I shall allow him ye ouerplus of one page or 2, in reuenge of you, but not reflecting upon you nor his bookes agt you. I know not well what wee shall driue at, but I haue receiued orders to study church-gouernemt. & a toleration & so to oppose Presbytery. (Stubbe to Hobbes, 7/17 October 1656; *CTH* 1:311–12)[5]

Stubbe evidently produced a draft of a book opposing Wallis's published thesis in which he attacked the Savilian professor for having "built ye whole fabrique upon Metaphors, of a flocke, family, body, &c. & alledged impertinent proofes of ye Apostles &c. wch are no more pertinent to us yn all ye other promises. & unlese hee proue ye [pre]sent ministry to bee Apostles, ye question about their commi[ssion] hath no more influence upon us yn ye debate betwixt Sigonius and Gruchius about ye power of [ye] Roman tribunes" (Stubbe to Hobbes, 25 October/4 November 1656; *CTH* 1:333).[6] The tract seems never to have

5. The reference to "du Moulin" is to Lewis du Moulin, son of the prominent French Huguenot Pierre du Moulin; he was appointed Camden professor of history in 1648 and supported the Independent cause with his *Paraenesis ad aedificatores imperii in imperio*. Owen's defense of Independency is his treatise *Of Schisme*, which attempts to argue that the Independent model of church polity does not invite schism in the church. It is unclear what "under hand abuse" Owen may have suffered at Wallis's hands, but Stubbe reports in a letter to Hobbes from 25 October/4 November 1656 that "in the 42d page D:r W hath touched him to ye quicke" (*CTH* 1:334). This refers to Wallis's denunciation of those who intolerantly insist upon formalities, especially at a university.

6. Noel Malcolm reports that Stubbe's reference to "Sigonius and Gruchius" is to a long-running dispute between the scholars Nicolas de Grouchy and Carlo Sigonio over the role of Roman tribunes (*CTH* 1:336 n. 10).

been published. In fact, it was apparently too harsh for Owen to permit its printing, although Stubbe reports to Hobbes that "D:^{r} Owen hath approued of my piece ag^{t} [Wallis's] *thesis,* with all y^{e} harsh language agt y^{e} Synod, I know not when it will bee printed" (Stubbe to Hobbes, 17/27 March 1657; *CTH* 1:456). Cawdrey, however, claims that "when Doctor *Wallis's Thesis* . . . was since printed, this Reverend Doctor did imploy, or at least encourage (an *Amanuensis* of his) Mr. Stubbs of Christ-Church (now advocate for Mr. Hobs) to write against it: Though indeed, when that work written, was found a *Scurrilous ridiculous piece* (for so I heare, he is since pleased to style it) he did not thinke it fit to be made publick, because (they were his own words) *'he would not have that cause suffer so much as to be defended by such a Penne'*" (Cawdrey 1658, 129–30).

Hobbes accepted Stubbe's invitation to attack Wallis by "instanceing at y^{e} absurdity of one argument, or y^{e} tendency y^{t} it hath to confusion, or y^{e} erecting a power beyond y^{e} Papall in jurisdiction," although he did not, as far as currently available evidence attests, cooperate with his suggestion to "penne a short letter censuring y^{e} tract," which would have appeared as part of Stubbe's published refutation of Wallis's *Mens Sobria* (Stubbe to Hobbes, 25 October/4 November 1656; *CTH* 1:337). Rather than attack Wallis in an anonymous letter appended to Stubbe's polemic, Hobbes incorporated a critique of the "Scottish Church-Politicks of Dr. Wallis" into the *Στίγμαι* of 1657.

The brief but substantive critique of Wallis's Presbyterianism in *Στίγμαι* is based on key elements in Hobbes's political philosophy, specifically his views on what constitutes a church and the relationship between ministerial and civil power. It should be clear that Presbyterianism is antithetical to Hobbes's principles because it asserts the autonomy of the church and its exemption from the authority of the civil sovereign. Independency, in contrast, has a Hobbesian warrant, since it holds (to use Wallis's words) "that each single Congregation, voluntarily agreeing to make themselves a *Church,* and chuse their own Officers, [are] of themselves *Independant,* and not accountable to any other *Ecclesiastical Government;* but only to the Civil Magistrate, as to the Publick Peace" (Wallis 1970, 34). Because maintenance of the public peace is the sine qua non of Hobbesian political theory, it can accommodate Independency fairly easily. Moreover, because the Independents made no claim to exercise quasi-judicial powers over the populace, they did not attempt to lessen the authority of the civil sovereign by claiming the right to regulate the spiritual affairs of the commonwealth.

Hobbes's *Στίγμαι* levels several specific charges against the doctrines in Wallis's thesis of 1654. The first of these is that Wallis's definition of the Christian ministry illegitimately separates the church from the state. We saw that Wallis defines ministers of the Gospel as "those of whom the preaching of the Gospel is demanded, ex officio, by Christ" (*Mens Sobria* 136). Hobbes responds to this definition by asking "what do you mean by saying preaching *ex Officio* is *enjoyned by Christ*? Are they Preachers *ex Officio,* and afterwards enjoyned to Preach? *Ex Officio* adds nothing to the definition; but a man may easily see your purpose to disjoyn your self from the State by inserting it" (*Στίγμαι* 3; *EW* 7:395). The root of Hobbes's objection here is, of course, that such a definition of a minister is founded upon the error of taking the present (Presbyterian) church for the Kingdom of God, from which it follows that "there ought to be some one Man, or Assembly, by whose mouth our Savior (now in heaven) speaketh, giveth law, and which representeth his Person to all Christians" (*L* 4.44, 335; *EW* 3:606). Since Wallis and other Presbyterians deny that the ordination or recognition of ministers of the Gospel is a matter subject to the jurisdiction of the civil sovereign, they in effect arrogate this part of sovereign authority to themselves.

A second, and more interesting, objection is the charge that Wallis's principles provide no way to determine who is a minister of the Gospel. Hobbes takes it for granted that Christ himself does not immediately speak to his ministers and invest them in their offices. The bestowing of ministerial office must therefore come through the authority of some human intermediary such as a bishop or another minister. But in such a case there is no way to verify that this person's authority is legitimate, since it is impossible to tell whether a person who claims authority to invest ministers in their offices was indeed granted such authority immediately from Christ. Furthermore, the attempt to trace the line of transmission of Christ's authority back to the time of the Apostles immediately runs into difficulty because going back "but a mater of sixscore years, you will find your Authority derived from the *Pope;* which words have a sound very unlike to the voices of the Laws of England" (*Στίγμαι* 3; *EW* 7:395). The tradition holds that the author-

God's ambassadors; pretending to have a right from God to govern every one his parish, and their assembly the whole nation" (*Behemoth* 1; *EW* 6:167). Later in the same work Hobbes complains that "Presbyterians are everywhere the same: they would fain be absolute governors of all they converse with, having nothing to plead for it, but that where they reign, it is God that reigns, and nowhere else" (*Behemoth* 4; *EW* 6:372–73).

ity to invest ministers of the Gospel in their offices arises from the successive "imposition of hands" back to the age of the Apostles, but Hobbes takes such an opinion as "too rude to be endured in a state that would live in peace" inasmuch as it separates ecclesiastical authority from that of the civil sovereign. Hobbes concludes "you can never prove you are a Minister, but by the Supream Authority of the Common-wealth" (*Στίγμαι* 3; *EW* 7:395).

Because the office of the ministry depends upon the authority of the civil sovereign, Hobbes reasons that the powers pertaining to that office likewise depend upon the will of the sovereign. In a passage guaranteed to anger Wallis, Hobbes remarks:

> You can have no other power then that which is limited in your Orders, nor that neither longer than [the sovereign] thinks fit. For if he give it you for the instruction of his subjects in their duty, he may take it from you again whensoever he shall see you instruct them with undutiful and seditious principles. And if the Soveraign power give me command (though without the ceremony of imposition of hands) to teach the Doctrine of my *Leviathan* in the Pulpit, why am I not if my doctrine and life be as good as yours, a Minister as well as you, and as publick a person as you are? (*Στίγμαι* 3; *EW* 7:396–97)

Hobbes thereby reaches a conclusion familiar from his political theory: the civil sovereign is God's representative on earth, and that those who claim an authority directly from God and contrary to that of the civil power are agents of the "Kingdome of Darknesse" who conspire to make citizens ignore their principal civic duty.

To Wallis's suggestion that the ministers of the Gospel should, on their own authority, institute public preaching of the Gospel at public gatherings such as market days, Hobbes replies that such a practice can only be part of a plan for lessening public support for the civil sovereign. Proponents of such a scheme, he writes, do not know "that many teachers unlesse they can agree better, do any thing else but prepare men for faction, nay, rather you know it well enough; but it conduces to your end upon the Market-dayes to dispose at once both Town and Country, under a false pretense of obedience to God, to a Neglecting of the Commandments of the Civil Soveraign, and make the Subject to be wholly ruled by your selves" (*Στίγμαι* 3; *EW* 7:397).

Wallis's response to these criticisms of his Presbyterian church poli-

7.1.2 *The Allegation of Atheism*

Both Wallis and Ward publicly accused Hobbes of being an atheist, notwithstanding the fact that in *Leviathan* and other works he offers proofs of the existence of God and makes no explicit avowal of skepticism about the existence of a deity. There is nothing novel in this accusation. As I mentioned in chapter 2, the publication of *Leviathan* unleashed a torrent of anti-Hobbes literature, and a standard theme in these polemics was that of Hobbes's alleged atheism. Mintz has described atheism as the central issue around which Hobbes's otherwise diverse opponents could unite: "It was atheism then which was at the heart of the controversy about Hobbes, the source of all the fears and seething indignation which Hobbes's thought inspired, the single charge which is most persistently made, and to which all other differences between Hobbes and his contemporaries can be reduced" (Mintz 1962, 45). Although it is going too far to claim that all of Hobbes's differences with his contemporaries can be "reduced" to the question of atheism, it is certainly true that the allegation of atheism plays a leading role in many attacks on Hobbes. I cannot treat all of the different authors who detected atheistic tendencies in Hobbes's work and will confine my attention to the writings of Wallis and Ward. As it happens, Wallis's charge of atheism is closely connected with Ward's, and these two together offer something of a joint critique of Hobbes's religious opinions. After examining the arguments used by Ward and Wallis to support the charge of atheism, and considering Hobbes's defense against the charge, I will argue that, on balance, the preponderance of the evidence shows that Hobbes probably was an atheist.

Before we begin this investigation, it is worthwhile remarking that *atheist* was a general term of abuse in the seventeenth century. Consequently, those who accused others of atheism did not always literally think that their opponents denied the existence of God. Rather, the epithet was often applied quite indiscriminately, so that its use may indicate only that the alleged atheist's religious views are in some way objectionable. Because the charge of atheism was made so frequently in seventeenth-century England, even though open profession of atheism was outlawed, it is difficult to tell how widespread atheistic views may have been in Hobbes's day.[8] The sense of the term *atheism* that I

8. See Aylmer 1978 for an attempt to gauge the extent of unbelief in seventeenth-century England.

take to be central to this question is the root sense in which atheism is the outright disbelief in God or gods. A. P. Martinich has drawn attention to free use of the term *atheism* in the seventeenth century and has reminded us that Hobbes's opponents' labeling him an atheist does not, by itself, go very far toward showing that he is an atheist in the strict or literal sense (Martinich 1992, 19–22). Nevertheless, Hobbes publicly defended a number of views that were widely regarded as underwriting, if not actually implying, atheism in the strict sense. Michael Hunter has remarked that in the context of seventeenth-century thought

> atheists were seen as people who denied the existence of God, either directly or by implication. It was axiomatic that unbelief would be sustained by views—usually materialistic—of a natural world that had originated without a beneficent creator and in which God's activity was limited or completely absent. But in addition, numerous other arguments were seen as part of the atheists' armory: a denial of the immortality of the soul and of any absolute morality; a skepticism about the text of the Bible, based either on its internal inconsistencies or on the supposed irreconcilability of Holy Writ with pagan history; or the opinion—which orthodox polemicists repeatedly tried to turn back on itself— that religion had first been introduced as "a *meer politick Contrivance*." (Hunter 1990, 441)

Hobbes, it should be noted, subscribed to all of these "atheistic" views. He championed materialism, rejected providential history, denied the immortality of the soul, described a state of nature devoid of the usual moral absolutes, challenged the authenticity of much of Scripture, and defined religion as "*Feare* of power invisible, feigned by the mind, or imagined from tales publiquely allowed" (*L* 1.6, 26; *EW* 3:45). As we will see, a great deal depends on whether Hobbes can maintain such views without being an "atheist by consequence," i.e., one who endorses principles that imply atheism even if he never explicitly declares disbelief in God. An atheist by consequence must contradict himself whenever he avows belief in God, and there are many apparent contradictions between Hobbes's principles and his professions of Christian belief. This makes it at least minimally plausible that Hobbes was sincerely committed to an atheistic worldview and intended his expressions of religious belief ironically or disingenuously. Whether we can ultimately maintain such an "ironist" reading of Hobbes's religious pronouncements will be taken up shortly, but we

sent by the goddess Hecate to frighten travelers or warn of impending evil. Empusa had the power to change forms but most frequently appeared with one leg of brass and the other of an ass—which Hobbes's compares with the "firm foot" of Scripture and the "rotten foot" of school divinity. Hobbes holds that no better exorcism can be found for the Empusa of school divinity than if "the rules of religion, that is, the rules of honoring and worshiping God, which are to be taken from the laws, are distinguished from the rules of philosophy, that is, the opinions of private men; and those things due to religion are granted to the Sacred Scripture, and those due to philosophy are granted to natural reason" (*DCo* epistle; *OL* 1:sig. h5v–h6r). Given Hobbes's absolutist theory of sovereignty, the task of honoring God therefore becomes no more than that of obeying the sovereign's decrees regarding worship. Hobbes further holds that once he has established the principles of true philosophy, the Empusa of school divinity will be dispatched, not by some sort of battle, but rather by letting in the light and frightening her away. These declarations against school divinity naturally annoyed both Ward and Wallis, who (as doctors of divinity) took them as attacks on Christianity itself.

In the *Elenchus* Wallis followed Ward's lead and imputed a contempt for all things religious and scriptural to Hobbes. In the epistle addressed to Owen he remarks that "with what arrogance *[ὑπερηφανεία]* and imperiousness *[pro imperio satis]* he tramples on all things both human and divine, writing terrible and horrible things of God, of sin, of the Holy Scripture, of all incorporeal substances in general, of the immortal soul of man, and of other weighty points of religion . . . is more to be lamented than doubted" (*Elenchus* epistle sig. A2). Wallis later elaborates on Ward's accusations in the context of commenting on a curious declaration in chapter 18 of *De Corpore*, where Hobbes remarks on the supposed success of his rectification of parabolic arcs. Hobbes there claims that "there are those who think that there is an equality between a right line and curve, but that it cannot now be found; now, they say, after the fall of Adam, without the special aid of divine grace" (Hobbes 1655, 161). Wallis takes these references to divine grace and Adam's fall as expressing Hobbes's disbelief in Scripture, and accuses him of making light of serious matters: "I realize how sceptically and sarcastically *[σαρκαστικῶς]* these things are said of divine grace. Indeed you boast that by your efforts you have found what others were expecting could not be found without 'the special aid of divine grace'" (*Elenchus* 89). Wallis adds that no right-thinking person could seriously doubt that the intellectual powers of man were

diminished after Adam's fall, and that there is all the more reason to think that divine grace is necessary for any successful inquiry into nature. He concludes that Hobbes is implicitly denying the role of divine grace in the acquisition of knowledge, so that Hobbes's boasts of his abilities amount to the atheistic tenet that knowledge of the world can be obtained without God.[9]

Not content with objecting to Hobbes's comments on the role of divine grace in mathematics, Wallis launched an attack on the atheistic tendencies implicit in the political theory and metaphysics of the Malmesburian philosopher. The essentials of this charge are contained in the following long passage:

> But perhaps you take the whole story of Adam's fall for a fable, and smile ironically at its introduction. Which indeed need not to be marveled at in you, since you want that "Religion, that is, the rules of honoring and worshiping God are to be taken from the laws," and you openly hope that "by exorcism this Empusa" (the Christian religion) "is frightened off and driven away." Nor is this discordant with what you have elsewhere concerning the origin of the world, namely that it is not by the force of argu-

9. Hobbes's rebuttal is to emphasize that it was allegedly *special* grace that was necessary to find the equality between a line and curve, whereas he claims only to have discovered it by ordinary means. This, he insists, is consistent with holding that there is some need of divine assistance in developing any science, so that Wallis, "taking no notice of the word *Speciall*, would have men think I held, that humane Sciences might be acquired without any help of God" (*SL 5*; *EW* 7:320). Hobbes cites the Jesuit Antonius Lalovera as holding that special grace is needed for this part of mathematics, because in the prolegomena to his treatise *Quadratura circuli et hyperbolae* Lalovera declares that "although the quadrature of the circle is possible in its nature, theologians have nevertheless thought fit to inquire whether in these days, that is after the fall of Adam, man can attain knowledge of this matter without the aid of special divine grace. And they have declared that this truth is so shrouded in darkness that none can see it unless the clouds of ignorance flowing from the prevarication of our first parents be dissipated by a ray of divine light, which opinion I judge to be most true" (Lalovera 1651, 12–13). Hobbes finds that "he (supposing he had found that Quadrature) would have us believe it was not by the ordinary and Naturall help of God (whereby one man reasoneth, judgeth and remembereth better then another) but by a Special (which must be Super[-na]turall) help of God, that he hath given to him of the order of *Jesus* above others that have attempted the same in vain. Insinuating thereby, as handsomely as he could, a Speciall love of God towards the *Jesuites*" (*SL 5*; *EW* 7:320). This reply may permit Hobbes to avoid Wallis's specific allegation, but it does little to diminish the appearance of arrogance in his pronouncements on his quadratures, nor does it unequivocally commit Hobbes to the view that divine assistance (albeit not special grace) is necessary for science. Hobbes removed the offending passage in the 1656 English version of *De Corpore* and it did not appear in the 1668 Amsterdam *Opera*.

ments or reasons that this matter is to be resolved, but that it is to be determined by the sovereign *[magistratus]:* as if this were not sufficiently agreed in the Holy Scripture *[codex]*, but should depend entirely on the suffrage of sovereigns whether or not the world ever had a beginning. But to what end do I mention Holy Scripture? This Scripture you may happen to observe in religion, and the Christian religion itself: that is, that you concede as much authority to it as is granted by the civil sovereign, and otherwise you wish all such authority to be removed, as if it belonged to the sovereign, not only whether or not the world once had a beginning, but also whether or not the sacred Bible is the word of God. But you appear to be no more concerned with God Himself than with the divine word, since you appear, I think, easily ready to set Him aside. You take it to be ridiculous, and what could never be conceived by any imagination, that anything should ever exist that is not a body; and you also hold (as in the end of this treatise) that all "incorporeal substances" are to be dismissed as the "inane words of the Scholastics." Who does not see that thereby you not only deny (and not just in words *[tantum non ῥητῶς]*) angels and immortal souls, but the great and good God himself; and if you were not wary of the laws (which to you is the highest "rule of honoring and worshiping God") you would profess this openly. And however much you may mention God and the Holy Scriptures now and again (although I do not recall your mentioning the immortal soul), it is nevertheless to be doubted whether you do this ironically and for the sake of appearance rather than seriously and from conviction. (*Elenchus* 89–90)

These charges summarize essentially the whole case against Hobbes's religious opinions and it is worthwhile to examine them somewhat more deeply.

The accusation that Hobbes makes the question of the eternity of the world depend upon the will of the sovereign picks up on declarations in part 4 of *De Corpore* to the effect that, because man's knowledge is confined to what is finite, there can be no demonstrative knowledge of whether the world is eternal or whether it had a beginning.[10]

10. Hobbes reasons:

Whatever we men know we take from our phantasms; but there is no phantasm of an infinite, whether of magnitude or time; nor can a man or any other thing have any conception of the infinite, beyond that it is infinite; nor if someone

Hobbes concludes "questions of the magnitude and origin of the world are therefore not to be settled by philosophers, but by those who are lawfully responsible for regulating the worship of God" (*DCo* 4.26.1; *OL* 1:336). This is actually Hobbes's reply to an argument in Ward's *Philosophical Essay.* Ward had undertaken to prove that the past must be finite because the supposition of an infinite past yields a contradiction (specifically, the number of generations from the beginning of the world to Abraham is equal to that from the beginning of the world to his great-grandson Joseph).[11] Hobbes reconstructed Ward's reasoning in *De Corpore* (albeit without naming him) and compared it to that of someone who concludes that there must be as many even numbers as natural numbers because there are infinitely many of both; he then asked whether "those who thus do away with the eternity of the world do not by the same argument do away with the eternity of the creator of the world?" (*DCo* 4.26.1; *OL* 1:337). Although Wallis may be somewhat wide of the mark when he characterizes Hobbes as holding that the question of the world's beginning is to be resolved by the sovereign without reference to the Scriptures, it is evident that Hobbes's comments constitute a skeptical response to any "cosmological" argument for the existence of God, in which God is identified as the first cause of the world.[12]

should ascend by right reasoning from any effect to its immediate cause, and thence to a further cause, and so on perpetually, yet he will not be able to proceed eternally, but wearied will at some point give up, not knowing whether he could have gone further. Nor will anything absurd follow, whether the world is agreed to be finite or infinite; since whether the creator of the world had determined it to be one way or the other, the same things that now appear could still appear. (*DCo* 4.26.1; *OL* 1:336)

11. The argument appears at Ward 1652, 14–17. See Probst 1993 for a discussion of the argument and Hobbes's response.

12. This particular point will be of interest shortly, when we consider Hobbes's supposed proofs for the existence of God. It is also worth noting that in his reply to Wallis on this point, Hobbes steers away from the question whether God's existence can be known and back to questions of church government. "Lastly, what an absurd question it is to ask me whether it be in the Power of the Magistrate, whether the world be eternall or not? It were fit you knew tis in the Power of the Supreme Magistrate to make a Law for the punishment of them that shall pronounce publiquely of that question any thing contrary to what the Law hath once pronounced. The truth is, you are content that the Papall power be cut off, and declaimed against as much as any man will; but the Ecclesiasticall Power which of late was aimed at by the Clergy here, being a part thereof, every violence done to the Papall Power is sensible to them yet; like that which I have heard say of a man, whose leg being cut off for the prevention of a Gangrene that began in his Toe, would nevertheless complain of a pain in his Toe, when his leg was cut off" (*SL* 6; *EW* 7:352).

When Wallis refers to Hobbes's denial of God, angels, and immortal souls by rejecting the doctrine of immaterial substances, he has in mind some of the closing reflections of *De Corpore,* where (after duly acknowledging that natural philosophy must depend upon hypotheses, and is therefore not wholly demonstrable) Hobbes announces that

> nevertheless, since I have assumed no hypothesis that is not possible and easily comprehended and I have reasoned legitimately from these assumptions, I have demonstrated that they may be [the true causes of the phenomena], which is the end of physical contemplation. If, having assumed other hypotheses, someone else should demonstrate the same or greater things, we shall owe him more thanks than I judge due to myself, provided that the hypotheses employed are conceivable. For as for those who say that anything is moved or produced by itself, by species, by power, by substantial form, by incorporeal substance, by instinct, by antiperistasis, by antipathy, by sympathy, by occult quality, and the other inane words of the Scholastics, these are all said to no purpose. (*DCo* 4.30.15; *OL* 1:431)

The picture here of a physics that proceeds from purely materialistic principles, unencumbered by reference to God or immaterial beings, fits perfectly the seventeenth-century image of the scientific atheist. It is therefore no great wonder that Wallis should take these comments as reflecting Hobbes's irreligious outlook.

Publication of the *Elenchus* was hardly the end of the questions regarding Hobbes's religion. Ward pursued his own interpretation of the atheistic tendencies in Hobbesian philosophy, and Hobbes took pains to deny the allegations raised by both Wallis and Ward; the result is that there were several exchanges over the question of Hobbes's religion and his sincerity in professing Christianity. Ward's most extensive criticism of Hobbes's religion appeared in the sixth and final section of his *In Thomae Hobbii Philosophiam Exercitatio Epistolica* and follows the lines of attack used by Wallis. Ward set himself the task of inquiring whether Hobbes's theories are friendly or hostile to Christianity (*Exercitatio* 313–22) and then of addressing the broader issue of Hobbes's attitude toward religion in general (*Exercitatio* 332–40). He naturally concluded that Hobbesian political and metaphysical doctrines are inimical to Christianity in particular and hostile to religion in general. The details of this critique can be left aside, but a brief summary of Ward's arguments is useful for assessing the evidence for the charge that Hobbes was an atheist.

nyed the true God in effect, as much as if he had done it with his lips" (*L* 3.42, 271; *EW* 3:493). Ward responded to all of this with apoplectic fury, likening Hobbes's theory to the outright denial of Christ and the martyrs, and assuring the reader that the only sane reaction to this theory was to lament that such odious nonsense had been printed.

The Hobbesian approach to Scripture left Ward with his "limbs numb with horror and indignation" (*Exercitatio* 324). Because *Leviathan* contains an extended argument to show that there is no scriptural authority for opinions that tend to diminish the power of the civil sovereign, Ward reads Hobbes as implying that neither Christ nor the Apostles could have done or demanded of their followers anything contrary to the will of the sovereign. In particular, Hobbes seems left with the implication that the saints and martyrs of the early church behaved in an un-Christian manner to the extent that their actions conflicted with the dictates of their respective sovereigns. Ward also takes issue with chapters 33 ("Of the Number, Antiquity, Scope, Authority, and Interpreters of the Books of Holy SCRIPTURE"), 36 ("Of the WORD of GOD, and of PROPHETS"), and 37 ("Of MIRACLES, and their Use") because he sees them as denying the authenticity and veracity of the Scriptures.

Like Wallis, Ward also detected atheistic tendencies in Hobbes's materialism. This is hardly an unexpected result because, as we have seen, materialism and atheism were closely identified in the seventeenth century. Hobbes's efforts to reconcile his materialism with Christian doctrine in *Leviathan* were unpersuasive to Ward, who declared that "nothing is more monstrous, nothing more unbecoming to a sane man than to wish to intrude this carnal and quasi-Muslim theory in place of Christianity" (*Exercitatio* 329–30). Ward even notes that, by denying all corporeal attributes (such as figure, motion, and place) to God while maintaining that the only substance is corporeal, Hobbes himself effectively makes it impossible to understand how God can exist (*Exercitatio* 340).

Hobbes's response to all such charges was a vehement denial. In *De Corpore,* for example, he attacks Ward ("Vindex") for suggesting in the *Vindicae Academiarum* that anything in the philosophy of *Leviathan* is contrary to religion.[14] Whenever pressed, Hobbes always

14. In particular, Hobbes denounces Vindex for having publicly charged him with being "irascible, a plagiarist, and an enemy to religion," on no further evidence than having heard rumors to this effect, which Hobbes considers as having been done "I should not say stupidly, but (although there is no vice without stupidity) rather with heinous wickedness *[sceleratè]*" (Hobbes 1655, 174).

claimed scriptural warrant for his religious doctrines, and he accused both Ward and Wallis of opposing them for purely selfish motives, i.e., in the interest of increasing their own power as doctors of divinity at the expense of the civil sovereign.[15] In the *Six Lessons*, he addressed both of the Savilian professors and cautioned them to "[t]ake heed of calling them all Atheists that have read and approved my *Leviathan*. Do you think I can be an Atheist and not know it? Or knowing it durst have offered my Atheism to the Press? Or do you think him an Atheist, or a contemner of the Holy Scripture, that sayeth nothing of the Deity, but what he proveth by the Scripture?" (*SL* 6; *EW* 7:350). In *Mr. Hobbes Considered* of 1662, he went on to explain, referring to himself in the third person, that his conception of a material God is in no way contrary to Scripture:

> It is by all Christians confest, that God is *incomprehensible;* that is to say, that there is nothing can arise in our fancy from the naming of him, to resemble him either in *shape, colour, stature,* or *nature;* there is no Idea of him; he is like nothing that we can think on. What then ought we to say of him? What Attributes are to be given him, not speaking otherwise than we think, nor otherwise than is fit, by those who mean to honour him? None but such as Mr. *Hobbes* hath set down, namely, expressions of reverence, such as are in use amongst men for signs of Honour, and consequently signifie *Goodness, Greatness,* and *Happiness.* . . . This is the Doctrine that Mr. *Hobbes* hath written, both in his *Leviathan,* and in his Book *de Cive,* and when occasion serves, maintains. What kind of Attribute I pray you is *immaterial,* or *incorporeal substance?* Where do you find it in the Scripture? Whence came it hither, but from *Plato* and *Aristotle* Heathens, who mistook those thin Inhabitants of the brain they see in sleep, for so many *incorporeal* men; and yet allowed them motion, which is proper only to things *corporeal?* Do you think it

15. In one illustrative passage, Hobbes claims that his "making the King Judge of Doctrines to be preach'd or published, hath offended you both; so has also his Attributing to the Civil Soveraign all Power *Sacerdotal.*" Further, he concludes that both Wallis and Ward "were angry also for [Hobbes's] blaming the Scholastical Philosophers," by rejecting many nonsensical notions common to school divinity and "*for detecting, further than you thought fit, the fraud of the Roman Clergy.* Your dislike of his Divinity, was the least cause of your calling him Atheist" (*MHC*; *EW* 4:434–35). The point here is Hobbes's familiar claim that school divines have attempted to mask their ignorance by the fraudulent use of meaningless terms, all with the intention of usurping the powers of the civil authority.

an honour to God to be one of these? And would you learn Christianity from *Plato* and *Aristotle?* (*MHC; EW* 4:426–27)

The success of Hobbes's defense against any charge of atheism therefore depends in large measure upon the extent to which his interpretation of Scripture can be taken seriously. But, as we will see, this is a matter of controversy.

As a further part of his defense against the charge of atheism, Hobbes even tried to counterattack by turning the allegation of against Wallis and Ward. He suggested (perhaps ironically) that the Savilian professors were so ready to accuse others of closet atheism because they themselves have grave doubts about the existence of God. As a result, he asks whether "finding your doubts of the Deity more frequent than other men do, you are thereby the apter to fall upon that kinde of reproach?" (*SL* 6; *EW* 7:353). Such a strategy had little effect and need not be an object of our attention.

It should be evident that in claiming that the doctors of divinity attack him for purely selfish motives Hobbes voiced a profound anticlericalism, or general hostility to the clergy. Although anticlerical attitudes are certainly common among atheists, atheism and anticlericalism are conceptually distinct and it would be a mistake to conclude that Hobbes's expressions of anticlerical sentiment in themselves constitute strong evidence of atheism. In Hobbes's day Protestant radicals (who were certainly not atheists) frequently voiced anticlerical attitudes, although these were typically based in the belief that the established church had departed from its true mission and its clergy had become corrupted by various "ungodly" influences. Hobbes's anticlericalism differs from that of such radicals and derives from his political theory, more specifically from his concern that the legitimate powers of the sovereign may be undermined by ambitious doctors of divinity. In any event, we cannot conclude that Hobbes was an atheist simply from the fact that he was hostile to the clergy.

Thus far the textual basis for any imputation of atheism to Hobbes is hardly overwhelming. He was certainly not the proponent of mainstream religious views, and his detailed and highly controversial readings of Scripture were not the sort of thing one generally finds in the theological literature of the period. It is evident that Hobbes rejected much in the standard interpretation of the Christian Scriptures, but it is not immediately obvious that he was attempting to signal his disbelief in the whole theological enterprise. When Hobbes reads such key scriptural terms as *angel* or *spirit* as denoting bodies, he may genuinely

(*L* 3.33, 200; *EW* 3:368), and that the books of the Old Testament were for the most part assembled long after the events they describe. Perhaps more tellingly, his account of the doctrine of the Trinity yields the remarkable result that Moses, Jesus, and the Apostles (together with their successors) are the three coequal persons of the Trinity. The doctrine depends upon Hobbes's understanding of the term *person*, which he defines as someone "*whose words or actions are considered, either as his own, or as representing the words and actions of another man, or of any other thing to whom they are attributed, whether Truly or by Fiction*" (*L* 1.16, 80; *EW* 3:147). God is therefore one person when represented by Moses, a distinct person when represented by Jesus, and still another when represented by the Apostles or their successors.[18] In the words of Edwin M. Curley, this theory "has certain disadvantages" when considered from the point of view of orthodox Christianity: it makes Moses, Jesus, and the Apostles coequal while denying the eternity of the three persons of the Trinity, and it hardly seems to restrict the trinity to just three members (Curley 1996a, 266).

Hobbes's accounts of prophecy and revelation are, if anything, more problematic for the Christian cause than his idiosyncratic theory of the Trinity. Someone skeptical of revealed religion typically argues that there can be no reliable criterion to determine whether God has communicated directly with any individual who claims to have had such communication. Hobbes explicitly endorses skepticism on this point in chapter 32 of *Leviathan*, when he observes that "How God speaketh to a man immediately, may be understood by those well enough, to whom he hath so spoken; but how the same should be understood by another, is hard, if not impossible to know. For if a man pretend to me, that God hath spoken to him supernaturally, and immediately, and I make doubt of it, I cannot easily perceive what argument he can produce to oblige me to beleeve it" (*L* 3.32, 196; *EW* 3:361). He then goes on to argue that such apparent revelatory vehicles as

18. Hobbes declares: "For as Moses, and the High Priests were Gods Representative in the Old Testament; and our Saviour himselfe, as a man, during his abode on earth: So the Holy Ghost, that is to say, the Apostles and their successors, in the Office of Preaching and Teaching, that had received the Holy Spirit, have Represented him ever since" (*L* 3.42, 267–68; *EW* 3:486–87). Martinich considers this part of Hobbes's philosophy to be a sincere attempt to make sense out of a doctrine that is beset with difficulties. He concludes that "[d]ue to the logical difficulties that the doctrine of the Trinity presents, almost any theory will be either inconsistent or heretical. What Hobbes has offered is a sophisticated attempt to render the doctrine both consistent and orthodox, and nothing more could be asked of either a philosopher or dogmatic theologian" (1992, 207–8).

dreams, visions, voices, or inspiration all fail to provide reliable evidence of actual revelation. He concludes that "though God Almighty can speak to a man, by Dreams, Visions, Voice, and Inspiration; yet he obliges no man to beleeve he hath done so to him that pretends it; who (being a man) may erre, and (which is more) may lie" (*L* 3.32, 196; *EW* 3:362). Such declarations plainly imply that nobody has ever had sufficient reason to believe claims of another's supernatural contact with God, and they should therefore apply as much to Moses, Jesus, or the Apostles as to any contemporary enthusiast. However, this skepticism is suddenly placed in abeyance in chapter 36 of *Leviathan,* when Hobbes discusses the Scriptures and their many claims of God's speaking to his prophets. Hobbes admits that it is a difficult matter to understand how God (who presumably lacks the usual apparatus of speech) can be said literally to speak to his prophets, and concludes that the prophets of the Old Testament must have received God's word in dreams and visions (*L* 3.36, 226–27; *EW* 3:417). Moses and the supreme prophets of Israel present a special case, because God is supposed to have spoken to them in a more familiar fashion. Hobbes determines that "in what manner God spake to those Soveraign Prophets of the Old Testament, whose office it was to enquire of him, as it is not declared, so also is it not intelligible, otherwise than by a voyce" (*L* 3.36, 229; *EW* 3:420). Prophecies in the New Testament are handled more easily, since at that time "there was no Soveraign Prophet but our Saviour; who was both God that spake, and the Prophet to whom he spake" (*L* 3.36, 229; *EW* 3:420). It strikes me as obvious that Hobbes has no warrant for this special treatment of the prophetic declarations recorded in the Scriptures. To put the matter bluntly, Hobbes's doctrines cannot be consistently maintained by anyone who takes revealed religion seriously. If claims of revelation based on dreams, visions, and inspiration are unreliable, and if there is no intelligible sense in which God can literally speak to man, then Hobbes's own principles demand that the extraordinary claims made by the Old Testament prophets, Moses, and Jesus himself should not be taken seriously today. Moreover, such claims should not have been so taken when they were first propounded centuries ago.

Even Hobbes's proofs of the existence of God can be seen as undermining the cause of religion. The sort of argument he seems to favor involves reasoning back to a "first cause." Since the investigation of causes "must come to this thought at last, that there is some cause, whereof there is no former cause, but is eternall; which is it men call God" (*L* 1.11, 51; *EW* 3:92). But, read in the context of his claim

ence for the Church of England. Nevertheless, Aubrey's testimony has at least some force in making the case for Hobbes's religious skepticism, if not outright atheism.

Curley's complex and intriguing reading need not be reproduced in detail here, but the question of ironic intentions (if any) in Hobbes's religious writings does need to be taken up. The heart of his interpretation consists of five theses, namely:

> 1) that if we compare Spinoza's Theological-Political Treatise with Hobbes's theological-political treatise, i.e., with his Leviathan, on a variety of topics which they both discuss in the theological portions of their works—specifically on the topics of prophecy, miracles, and the authority of Scripture—we shall find quite a lot in Spinoza's work which Hobbes might have found to be bolder than what he had written on the same topics; 2) that where Spinoza's position is bolder, Hobbes' less radical position is often stated in a way suggesting irony; 3) that since irony can function both as a protective device and as a way of hinting at views one would hesitate to express openly, Hobbes' use of it is evidence that he would have gone further than he did in the direction of unorthodoxy, if the political situation had permitted him to do so safely; 4) that it is entirely credible that Hobbes said to Aubrey what Aubrey says he said; and finally 5) that Hobbes is properly viewed as a precursor of such Enlightenment figures as Voltaire and Hume, that in spite of the deference he often shows to orthodox Christian doctrines, he is essentially a secular thinker, whose religious views are subversive of those held by most Europeans of his time. (Curley 1992, 511–12)

A crucial piece of this approach is Curley's claim that Hobbes uses a rhetorical device he calls "suggestion by disavowal," wherein an author marshals evidence or argument that tend toward a certain conclusion, but then disavows the seemingly obvious conclusion. The purpose of suggestion by disavowal is clear enough: it provides a kind of cover in case one is later charged with expressing unsavory opinions, while at the same time it indicates the author's real intention. To take a simple but apposite example, consider Hobbes's claim that "If I should say I have heard that *Dr. Wallis* is esteemed at *Oxford* for a simple fellow, and much inferior to his fellow-professor *Dr. Ward* (as indeed I have heard, but do not believe it) though this be no great disgrace to *Dr. Wallis*, yet he would think I did him injury" (*SL* 6; *EW* 7:354). The obvious point of this remark is to disparage Wallis's intellectual abili-

tence. He replied in *Στίγμαι* that no more than two of the passages cited by Wallis were germane to the case, and that neither of these was a proper construction involving *adduco* (*Στίγμαι* 2; *EW* 7:391–93). To support his philological criticisms, Hobbes enlisted the help of Henry Stubbe, whose extensive classical learning was ideally suited to this kind of polemical encounter. He solicited a letter from Stubbe attacking Wallis's philological conclusions, and then printed it anonymously at the end of *Στίγμαι* under the pretense that it had been sent to him by an unnamed third party.[21] After an extensive (if utterly one-sided) analysis of the many grammatical and philological points in the dispute, Stubbe declared:

> And now I conceive enough hath been said to vindicate Mr. Hobbes, and to shew the insufferable ignorance of the puny professor, and unlearned Critick. If any more shall be thought necessary, I shall take the paines to collect more examples and Authorities, though I confess I had rather spend time otherwise then in matter of so little moment. As for some other passages in his book, I am no competent judge of Symbolick Stenography. (*Στίγμαι* appendix; *EW* 7:426–27)

Stubbe made good on his threat to "collect more examples and Authorities," and published them in 1657 under the title *Clamor, Rixa, Joci, Mendacia, Furta, Cachiny; or, a Severe Enquiry into the late Oneirocritica Published by John Wallis, Grammar-Reader in Oxon.* This piece was a rebuttal to Wallis's *Dispunctio* (which contained some disparaging remarks about Stubbe)[22] and included the text of the origi-

21. After some preliminary skirmishing on the propriety of *adducis malleum*, Hobbes writes, "Being come thus far I found a friend that hath eased me of this dispute; for he shewed me a letter written to himself from a learned man, that hath out of very good Authors collected enough to decide all the Grammatical questions between you and me both Greek and Latin. He would not let me know his name, nor any thing of him but only this, that he had better ornaments then to be willing to go clad abroad in the habit of a Grammarian" (*Στίγμαι* 2; *EW* 7:393). The pretense of the letter's being written to a friend of Hobbes is, of course, a sham, as is evident from Stubbe's letter to Hobbes of 26 November/6 December 1656. Stubbe there writes of an enclosure, "I here send yo^r^ a long letter; I doubt not but you can forgiue such faults . . . as are occasioned by excesse of zeale to serue yo^u^. . . . I leaue y^e^ busynesse wholly to yo^r^ management" (*CTH* 1:378).

22. In the course of the philological wrangling in *Dispunctio*, Wallis writes, "Now, enter your second, (with a Rod at his back,) what says hee? Hee says *'Tis ridiculous.* That's easyly sayd: But saying so, doth not make it so. Hee says that, *Hee had, he is sure, been whipped soundly in Westminster School, by the learned Master thereof at present.* . . . That the *present master* is a *learned* man . . . I shall easily grant; . . . But I perceive hee hath had the hap sometimes to breed up sawcy boys. But leaving this Epistoler to

nal letter as printed in Στίγμαι, together with Wallis's replies, and Stubbe's further responses. The emphasis the disputants placed on such seemingly trivial points of usage testifies to the powerful role seventeenth-century authors accorded to matters of language in the conduct of controversies such as this one. The war between Hobbes and Wallis involved weapons beside the logic and close argumentation characteristic of geometric proof, and one of the most important was that of eloquence.

Subtleties of classical grammar and style were not the only focus of attention in that part of the dispute that focused on questions of language. Each disputant also tried to show that his opponent's efforts were too crude to pass the test of true eloquence, either because his language was excessively harsh and vulgar or because his attempted witticisms fell flat. Thus, when Hobbes devoted the last of his *Six Lessons* to the manners of the Savilian professors, he found it useful to accuse them of not knowing how to conduct themselves in a public dispute: "It cannot be expected that there should be much Science of any kinde in a man that wanteth Judgement; nor Judgement in a man that knoweth not the Manners due to a publique disputation in writing" (*SL* 6; *EW* 7:331). Hobbes also had to argue that his own harsh language was consistent with good manners, which he did by invoking "Vespasian's Law," or the principle that good manners forbid one from initiating the use of bad language in a dispute, but there is no harm in replying in kind. Wallis responded with a catalogue of Hobbes's ill language and suggested that the reason he had switched from the Latin of *De Corpore* to the English of the *Six Lessons* was that "when ever you have thought it convenient to repaire to Billingsgate, to learn the art of Well-speaking, for the perfecting of your naturall Rhetorick; you have not found that any of the Oister-women could teach you to raile in Latin, and therefore it was requisite that you apply your selfe to such language as they could teach you" (*Due Correction* 2).[23] Such mutual accusations of bad language are of little interest in themselves, except to the extent that they testify to the importance the disputants attached to rhetorical form, but they are a recurring theme throughout the exchanges between Hobbes and Wallis.

Real eloquence requires wit as well as proper words, and Wallis attempted to evince both the breadth of his learning and the sharpness

ruminate when and where he was whipped last, and, for what offence (whether *Ignorance* or *Impudence*) let's hear what he hath to say to the businesse" (*Dispunctio* 20).

23. Billingsgate is that part of London where the fish markets were located. It was synonymous with rude language.

of his wit with a number of clever rhetorical tricks. One of these involves the rhetorical figure known as *paranomasia,* which is an extended wordplay based on the similar sounds of different words. Wallis uses this figure in the *Elenchus* as a joking way of showing that Hobbes himself is really the specter Empusa. He does this by concocting a derivation of *Hobbes* from *Empusa,* working his way through from Greek to English by construing *Empusa* as derived from ἕν (one) and ποῦς (foot), which he then connects to the child's game called "ludus Empusae" in ancient Rome. The game requires one child to hop about on one foot as long as possible but be beaten by his playmates if the other foot should touch the ground; thus we get *hop* from *Empusa,* which quickly turns into *Hobbes* as the hobgoblin Empusa is identified with the philosopher from Malmesbury.[24]

Hobbes denounced such attempted witticisms as "Levity and Scurrility" that comport poorly with the "gravity and sanctity requisite to the calling of the Ministry" (*SL* 6: *EW* 7:354–55). In particular, he dismisses the paranomasia on Empusa by asking, "When a stranger shall read this, and hoping to finde therein some witty conceit, shall with much adoe have gotten it interpreted and explained to him, what will he think of our Doctors of Divinity at *Oxford,* that will take so much pains as to go out of the language they set forth in, for so ridiculous a purpose?" (*SL* 6; *EW* 7:355). Stubbe lent his talents to the defense of Hobbes in this matter and examined at great length the philological basis for Wallis's "silly clinch, which will not passe for wit either at *Oxford,* or at *Cambridge;* no nor at *Westminster*" (Στίγμαι appendix; *EW* 7:410).

A final instance of Wallis's efforts to evince great wit through wordplay and questionable taste will close our study of the linguistic aspects of the dispute. This episode arose in connection with Wallis's criticisms of Hobbes's application of his "method of motions" in chapter 16 of *De Corpore,* which deals with accelerated motion (*DCo* 3.16.2; *OL* 1:185–86). Among many other criticisms, Wallis had complained that

24. In the original (which doesn't really translate well): "Erat enim & *Empusa tua,* Daemonium illud Atheniense, ex pede dignoscenda, quippe erat (uti ais) *pedibus altero aeneo, alter Asinino,* (& utro horum tu dignosci malis, penes te sit optio,) sed utut (uti videtur) duos pedes habuerit non tamen nisi uno incedebat, ut aliunde si opus sit discas (unde liquet ex eorum lemurum numero fuisse quos nos Anglicè dicimus *Hob-goblins,*) nempe ab ἕν & ποῦς factum est nomen illud: unde & puerorum ludus ille, *Empusae* ludus dictus (Anglicè *Fox, fox, come out of your hole,* hoc est, *heus vulpes de foveâ prodi*) nomen sortitur, quo puer ille qui *vulpes* audit, alter suspenso pede, altero subsultans incedit, (quod est Anglicè *to Hop,*) à reliquis flagellandus si utroque terram tetigerit" (*Elenchus* 4).

the formulation "in every uniform motion" should be taken as a plural "in all uniform motions," because the doctrine "does not concern single motions taken singly, but many motions taken together and compared with one another" (*Elenchus* 40). Expanding on this criticism in the *Due Correction*, he comments that "there must be at least two Motions, because two Times; unlesse you will say, that one and the *same motion* may be *now, and anon too*" (*Due Correction* 96). Hobbes correctly guessed that the expression "now, and anon too" was intended as some kind of joke. He wrote to Stubbe asking for help in tracking down the source of the quotation. Stubbe replied that "[i]t is not taken out of any ballad, but referres to Obadiah Sedgewicke, who haueing marryed for his first wife a chambermayde, presently after y^e^ marryage dinner was ended, hee took her aside into the draweing roome, and desired to anticipate those nuptiall pleasures hee was otherwise not to partake of till night. Y^e^ mayde desired him to forbeare till night; but he replyed, now and anon too. and thus you haue y^e^ story as it was related by Sedgewickes brother in lawe to mee y^e^ other day" (Stubbe to Hobbes, 29 November/9 December 1656; *CTH* 1:379). Armed with this information Hobbes then counterattacked against Wallis's charge that "the ribauldry in your obscene Poem *De Mirabilibus Pecci*" indicates the Malmesburian philosopher's utter lack of sophistication (*Due Correction* 3). Hobbes responded by charging that, at least in the matter of comparative obscenity, Wallis must be judged the more vulgar of the two disputants.[25]

7.3 LOYALTY, DUPLICITY, AND THE POLITICS OF THE RESTORATION

The final extramathematical topic at issue in the dispute is that of the political loyalties of the two principal disputants. We have already cov-

25. Hobbes writes: "For my verses of the Peak, though they be as ill in my opinion as I beleeve they are in yours, and made long since, yet are they not so obscene, as that they ought to be blamed by Dr. *Wallis*. I pray you Sir, whereas you have these words in your *Schoole-Discipline* page 96. *unlesse you will say that one and the same motion may be now, and anon too*; what was the reason you put these words *now and anon too* in a different Character, that makes them to be the more taken notice of; Do you think that the story of the Minister that uttered his affection (if it be not a slander) not unlawfully but unseasonably, is not known to others as well as to you? what need you then (when there was nothing that I had said could give the occasion) to use those words; there is nothing in my verses that do *olere hircum*, so much as this of yours. I know what good you can receive by ruminating on such Ideas, or cherishing of such thoughts" (*Στίγμαι* 2; *EW* 7:389).

litical connections, notwithstanding the fact that his holding of the office conflicted with the Savilian statutes.[27] Moreover, as a member of the Westminster Assembly and an active supporter of the Presbyterian cause, Wallis had taken the Solemn League and Covenant in 1643; he had also subscribed to the engagement in 1650, and—most damningly—he had assisted the Parliamentary forces during the Civil War by deciphering the king's correspondence captured at the Battle of Naseby. The king's pardon was readily forthcoming, however, and Parliament confirmed Wallis's two positions in the university. He was even admitted as one of the king's chaplains in ordinary and appointed in 1661 to the group of divines charged with revising the Book of Common Prayer.

Hobbes undoubtedly welcomed the Restoration because the reestablishment of the Stuart house suited his professed preference for monarchy over other forms of government. Nevertheless, the events of 1660 did not leave him altogether without grounds for concern. His return to England from France in 1651 was interpreted by many as an abandonment of the Royalist cause he had once supported. Furthermore, his reputation for atheism and his pronounced anticlericalism made him a favorite target for the denunciations of Anglican divines.[28] Shortly after the restoration of Charles II, Wallis publicly charged Hobbes with disloyalty to the monarchy. In 1662 the Savilian professor of geometry claimed that the events of recent years had made *Leviathan* "somewhat out of season," since the monarchy had been restored.

27. Aubrey reports that in 1657 Wallis "gott himselfe to be chosen (by unjust means) to be Custos Archivorum of the University of Oxon, at which time Dr. Zouch had the majority of voices, but because Dr. Zouch was a malignant, (as Dr. Wallis openly protested, and that he had talked against Oliver) he was putt aside. Now, for the Savilian Professor to hold another place besides, is so downeright against Sr. Hen. Savile's Statutes that nothing can be imagined more, and if he does, he is downright perjured. Yet the Dr. is allowed to keepe the other place still" (Aubrey 1898, 2:569). Anthony à Wood gives an equally damning assessment in his *Athenae Oxonienses*: "The famous Dr. Rich. Zouch, who had been an Assessor in the Chancellours Court for thirty years or more, and was well vers'd in the Statutes, Liberties, and Privileges of the University, did, upon great intreaties stand for the said place of Antiquary or Custos Archivorum thereof, but he being esteemed a Royalist, Dr. J. W. was put up and stood against him, tho altogether incapable of that place, because he was one of the Savilian Professors, a Cambridge man, and a stranger to the usages of the University. At length by some corruption, or at least connivance of the Vice Chancellour, and perjury of the Senior Proctor (Byfield), W. was pronounced elected" (Wood [1813] 1967, 2:cols. 414–15). This incident also led to Stubbe's rebuke of Wallis in *The Savilian Professours Case Stated* (Stubbe 1658).

28. As we saw in chapter 6, Hobbes had to face the possibility of being tried for heresy when the Restoration Parliament ordered an investigation of his works in 1666, an investigation eventually blocked by his patron Arlington.

More specifically, Wallis charged that *Leviathan* had been "written in Defense of *Olivers Title* (or whoever by whatsoever means can get to be upmost)," and that Hobbes's return to England in 1651 amounted to "deserting his *Royal Master* in distresse" (*HHT* 5).

The first of these two accusations of disloyalty is obvious nonsense and was easily shown to be so by Hobbes. When *Leviathan* was published in 1651 Oliver Cromwell was a general in the Parliamentary army. The office of Lord Protector was not established until late 1653, so (unless Hobbes had a rare prophetic gift) there is no plausible sense in which he could have written *Leviathan* in defense of Cromwell's claim to power.[29] Furthermore, because his political philosophy places obedience to lawful authority at the center of a citizen's obligations, Hobbes could hardly be charged with encouraging rebellion.

The accusation that Hobbes "deserted" the Royalist cause by returning to England in 1651 is somewhat more difficult to assess. Scholars have often puzzled over the question of why Hobbes should have returned, given that his previous association with the Royalist cause would have left him with few prospects under the new regime. Some have argued that Hobbes's actions show him to have been a supporter of the Commonwealth, and to have been an active participant in the engagement controversy, which centered on the question of whether and by what means submission to the newly established Commonwealth could be justified.[30] The "Review and Conclusion" to *Leviathan* contains a fairly direct acknowledgment of the question of the difficult political situation in England, and has been read as Hobbes's declaration of support for the "engagers" who advocated submission to the new government. Hobbes declares that *Leviathan* was "occasioned by the disorders of the present time," and that his intention was "to set before mens eyes the mutuall Relation between Protection and Obedience; of which the condition of Humane Nature, and the Laws Divine, (both Naturall and Positive) require an inviolable observation" (*L*, "Review and Conclusion," 395–96; *EW* 3:713).

The formula of "mutual relation between Protection and Obedi-

29. Hobbes makes this point as follows: "What was *Oliver* when that Book came forth? It was in 1650, and Mr. *Hobbes* returned before 1651. *Oliver* was then but General under your Masters of the Parliament, nor had yet cheated them of their usurped Power. For that was not done till two or three years after, in 1653, which neither he nor you could foresee" (*MHC*; *EW* 4:420).

30. See Metzger 1991, 131–35, for an overview of the controversy and Hobbes's place in it. The engagement that stands at the center of the controversy reads, "I do declare and promise that I will be true and faithful to the commonwealth of England, as it is now established, without a King or House of Lords" (Prall 1968, 238).

of human motivation and a cynical, self-serving theory of obligation) and took credit for having the philosopher driven from court (Metzger 1991, 92–97; Sommerville 1992, 24–25). Thus, there seems to be little basis for thinking that Hobbes abandoned the Royalist cause and sided with its enemies.

Hobbes's intentions in writing *Leviathan* (so far as they can be determined) also seem to avoid the charge of pure opportunism. It is true, as Skinner notes, that Hobbes boasted in 1656 that his doctrine "hath framed the minds of a thousand Gentlemen to a conscientious obedience to present Government, which otherwise would have wavered in that Point" (*SL* 6; *EW* 7:336). But this fact alone does not justify the inference that Hobbes wrote *Leviathan* for the purpose of legitimating the authority of the Commonwealth. Hobbes's own account was that during his exile he "staid about *Paris,* and had neither encouragement nor desire to return into *England,*" and he "wrote and published his *Leviathan,* far from the intention either of disadvantage to His Majesty, or to flatter *Oliver* (who was not made Protector til three or four years after) or purpose to make way for his return" (*MHC; EW* 4:415).

Royalist hopes were by no means extinguished during the period when Hobbes was writing *Leviathan* (1650–51). On the contrary, there was every expectation in the exiled court that Charles II could gain the throne by military means. The remaining Royalist forces were joined in an alliance with Scotland in 1650, which was concluded after the king acceded to Scottish demands that he commit himself to the establishment of Presbyterianism. In the course of events Charles II's attempted invasion of England came to grief in September of 1651 with his crushing defeat at the Battle of Worcester. At that point even the most optimistic Royalists were prepared to concede that the cause was lost, at least for the foreseeable future. In this context there is no reason to think that Hobbes wrote *Leviathan* with the intention of supporting the Commonwealth, since its main argument could just as easily have been intended to show the reluctant supporters of the vanquished Parliamentary cause that they owed allegiance and obedience to the crown. Hobbes himself suggests that *Leviathan* was a work that could have been put to use to defend either the king or his enemies. In the dedicatory epistle of the *Problemata Physica,* which was addressed to Charles II, Hobbes remarks that his opponents should not "turn it to a fault, if fighting against your enemies, and snatching up whatever weapons I could, I made use of a double-edged sword" (*PP* epistle; *OL* 4:303).

Hobbes actually welcomed Wallis's accusations of disloyalty, as they

gave him the opportunity to counterattack and try to salvage something from a dispute he was evidently losing badly. We saw in chapter 6 that by the time that Wallis's *Hobbius Heauton-timorumenos* appeared in 1662, Hobbes's standing as a man of science had been demolished by the continued refutation of his mathematical works. When Wallis began to raise questions of Hobbes's political loyalties, the philosopher from Malmesbury gladly compared his record with that of Wallis. He exulted that "Mr. *Hobbes* could long for nothing more than such an occasion to tell the world his own and your little stories, during the time of the late Rebellion" (*MHC; EW* 4:413). Hobbes assembled a catalog of Wallis's misdeeds in *Mr. Hobbes Considered,* which included siding with the Parliamentary forces against the king, deciphering the royal correspondence, acting with the Westminster Assembly to alter the form of church government without consent of the king, currying favor with Cromwell and his associates by dedicating his *Elenchus* to vice-chancellor Owen, and preaching principles that inspired rebellion. Hobbes concludes:

> Therefore of all the Crimes (the Great Crime not excepted) done in that Rebellion, you were guilty; you, I say, *Dr. Wallis,* (how little force or wit soever you contributed) for your good will to their cause. The King was hunted as a Partridge in the Mountains; and though the Hounds have been hang'd, yet the Hunters were as guilty as they, and deserved no less punishment. And the Decypherers, and all that blew the horn, are to be reckoned amongst the Hunters. Perhaps you would not have had the prey killed, but rather have kept it tame. And yet who can tell? I have read of few Kings deprived of their Power by their own Subjects, that have lived any long time after it, for reasons that every man is able to conjecture. (*MHC; EW* 4:419)

In actual fact neither Wallis nor Hobbes seems to have suffered from this airing of mutual charges of disloyalty. Wallis was not damaged by his Cromwellian past and continued to enjoy the favor of Charles II while he became a zealous Conformist to the Church of England. For his part, Hobbes was not abandoned by his friends or allies, and although he always had enemies at the court, Charles II seems to have regarded him affectionately. These charges of political intrigue and disloyalty thus function as a side issue in the dispute rather than a focal point.

CHAPTER EIGHT

Persistence in Error

Why Was Hobbes So Resolutely Wrong?

For all men by nature Reason alike, and well, when they have good principles. For who is so stupid, as both to mistake in Geometry, and also to persist in it, when another detects his error to him?

—Hobbes, *Leviathan*

The account assembled in the preceding chapters leads fairly naturally to a single conclusion: Hobbes was led by a misplaced faith in the efficacy of his materialistic foundations for geometry to think that he could quickly dispatch all the great problems of that science. For his part, Wallis undertook the refutation of Hobbesian geometry primarily for the purpose of discrediting Hobbes's "dangerous" metaphysical, theological, and political theories. The result was a bitter and very public dispute that dragged on for decades and encompassed a host of issues beyond technical details of circle quadrature.

Yet if this line of thought is pursued, there remains the problem of explaining why Hobbes should have been so persistent in his geometric errors. Why, after all, did he not simply admit that he was mistaken, cut his losses, and spare himself the profound humiliation that inevitably followed his repeated forays into the murky waters of circle quadrature? In short, there is a fundamental question that must loom large in any account of Hobbes's geometric endeavors: how did a man whose mathematical abilities were once ranked highly enough to earn him a reputation as one of Europe's mathematical cognoscenti end up filling hundreds of pages with miserably failed attempts at the solution of great geometric problems?

Making the minimal assumptions that most of what agents do makes sense to them as they do it and that they generally regard their beliefs as rationally grounded, we are left with the problem of ex-

plaining how Hobbes could have been engaged in such an apparently irrational project for such a long time.

I believe that Hobbes's conduct in the dispute with Wallis can be largely explained by appealing to the status of geometry within his system. Having insisted that all of philosophy must be grounded in the metaphysics of matter and motion, Hobbes readily accepted the consequence that geometry is an integral part of philosophy and that continued failure in geometric matters must be symptomatic of philosophical ineptitude. In other words, Wallis was right to characterize Hobbes as having "set such store by geometry as to hold that without it there is hardly anything sound that could be expected in philosophy" (*Elenchus* 108). He was also correct in his judgment that the most effective way to oppose Hobbes's philosophical principles was to "show how little he understands this mathematics (from which he takes his courage)" (Wallis to Huygens, 1/11 January 1659; *HOC* 2:296). Early on in the dispute, Hobbes was sensitive to the damage that Wallis could do to his reputation as a man of learning, and he tried to downplay the significance of his purely technical errors by insisting that

> [i]t is in Sciences as in Plants; Growth and Branching is but the Generation of the Root continued; nor is the Invention of Theoremes any thing else but the knowledge of the Construction of the Subject prosecuted. The unsoundness of the Branches are no prejudice to the Roots; nor the Faults of Theoremes to the Principles. And Active Principles will correct false Theoremes if the Reasoning be good; but no Logique in the world is good enough to draw evidence out of false or unactive Principles. (*SL* epistle; *EW* 7:188)

This pretense could not last. Hobbes had staked too much on his geometry to pretend that his failings were so many minor faults to be corrected by the rigorous application of his true principles; and even so he was still committed to rectifying his errors through the application of his "active principles" and thereby making good his claims to preeminence in geometry, i.e., actually squaring the circle.

Hobbes ultimately saw his entire philosophical program threatened by the prospect of his inability to deliver the great mathematical results he had promised. He concluded that it was no use to attempt to salvage a modicum of respectability by insisting that the intrinsic difficulty of the problems could excuse his failures. Hobbes's geometric errors were

generally errors of ignorance (although he would never publicly admit to such a fault), and he had in any case proclaimed that the superiority of his methodology should greatly advance the science of geometry by solving previously unsolved problems. To find the area of a circle is, on Hobbes's account of the matter, no more than to deduce the properties of something we construct. Thus, to suggest that a problem so simple to pose might nevertheless escape our efforts at solution would be tantamount to saying that the proper science of the commonwealth (itself an "artificial body" constructed by human agreement) might nevertheless fail to settle questions of right and obligation. Hobbes could simply not abide such a prospect.

After some hesitation in the 1650s and 1660s, Hobbes concluded that he had nothing to lose by rejecting the geometry of his opponents and maintaining the essential correctness of his procedures, no matter how wildly they may conflict with the received view of the matter. In this Hobbes's actions were much like those of a man who "throweth his goods into the Sea for *feare* the ship should sink" (*L* 2.21, 108; *EW* 3:197). Throwing classical geometry overboard was a serious price to pay, but Hobbes came to see it as the only way to avoid an even worse fate: the outright refutation of his philosophy as a whole. Whether this was ultimately a rational decision on Hobbes's part is a question I will leave to the judgment of the reader.

If anything else is clear about this dispute it is the fact that the claims of neither Hobbes nor Wallis can be properly characterized as a dispassionate search for the truth. Both participants' motives for pursuing the quarrel are doubtless complex, but it is beyond question that Wallis placed a high priority on discrediting Hobbes's metaphysical and theological doctrines, which he regarded as utterly pernicious. Furthermore, I take it that Hobbes's bitter enmity toward Wallis was the product of both personal and political factors: Wallis had done more damage to Hobbes's intellectual reputation than any other of his critics, and in any case the Savilian professor's standing as a "school divine" placed him squarely in opposition to Hobbesian political principles. There is consequently no need to doubt Hobbes's declaration that "I had never answered your Elenchus as proceeding from Dr. *Wallis*, if I had not considered you also as the Minister to execute the malice of that sort of people that are offended with my *Leviathan*" (*Στίγμαι* 1; *EW* 7:381).

8.1 HOBBESIAN GEOMETRY AND THE SOCIOLOGY OF SCIENTIFIC KNOWLEDGE

For all that there is an undeniable sociopolitical element to the controversy, I am convinced that it was not driven by strictly social or extramathematical factors. Hobbes was grievously mistaken about matters of great mathematical importance, and his mistakes in this science produced catastrophic consequences. There is, however, another way of thinking about the course of the dispute. Proponents of the "sociology of scientific knowledge" hold that scientific or mathematical controversy is merely symptomatic of "deeper" sociopolitical differences, and that the resolution of such controversy involves the triumph of one set of social interests over another. Thus, a sociologist of knowledge interested in the struggle between Hobbes and Wallis would hold that the *real* issues between them were those concerning religion and politics, and that mathematics mattered in the course of the quarrel only to the extent that it provided the contestants a convenient arena. Since the publication of Shapin and Schaffer's *Leviathan and the Air-Pump* (1985), this sort of analysis has become quite commonplace in Hobbes scholarship. Nevertheless, a consideration of the details of the present case shows that there is something quite seriously wrong with the attempt to see the controversy as driven purely by social factors. It will therefore be worthwhile to consider, by way of conclusion, the case against a purely sociological reading of Hobbes's dispute with Wallis.

Shapin and Schaffer enthusiastically endorse the thesis that scientific controversy is nothing more than a cover for more fundamental and important differences of opinion on how best to achieve social order. According to their analysis, scientific disputes are typically clashes between different "forms of life" or structures of rules and social conventions that dictate what knowledge is and how competing claims to knowledge are to be adjudicated. Moreover, these rival forms of life are linked to different political programs, and the resolution of a scientific controversy is achieved when one political program gains ascendancy and carries its affiliated scientific form of life in its wake. They inform us

> (1) that the solution to the problem of knowledge is political; it is predicated upon laying down rules and conventions of relations between men in the intellectual polity; (2) that the knowledge thus produced and authenticated becomes an element in political

(i.e., political) means, including the formation of strategic alliances with other social interest groups, Machiavellian strategies for outmaneuvering the enemy, brute force, and similar ploys.

This is not the place to pronounce upon the worth of Wittgenstein's later philosophy, but it is important to see just what follows from the equation of scientific (and mathematical) communities with forms of life. In particular, it is worthwhile asking whether this account of forms of life can underwrite an interpretation of the Hobbes-Wallis controversy grounded in strict sociological reductionism. What, then, is a form of life? As with most Wittgensteinian concepts, it is not easy to answer this question succinctly. A form of life is fundamentally a collective or communal concept: it is a system of rules and practices upon which members of a community agree, and conformity to which is required if one is to be a member of the community. A form of life is also the community's standard for what patterns of behavior and response will count as intelligible or coherent. As Kripke defines it in his discussion of rule-following: "The set of responses in which we agree, and the way they interweave with our activities, is our *form of life*. Beings who agreed in consistently giving bizarre . . . responses would share in another form of life. By definition, such another form of life would be bizarre and incomprehensible to us" (Kripke 1982, 96). The dictates of different forms of life thus constitute boundaries between mutually unintelligible communities, and there is no possibility of meaningful interaction across these frontiers. Thus, to employ the Wittgensteinian notion of a form of life in the analysis of scientific controversy, it must be shown that adherents of one research program find their opponents' claims not merely false but downright incomprehensible. To use the terminology derived from Kuhn's *Structure of Scientific Revolutions* (1970), the research programs would have to be incommensurable.[2] More specifically with regard to the Hobbes-Wallis dispute, the program of sociological reductionism would have to show that Hobbes and Wallis literally could not understand one another, and that they regarded each other's claims as flatly unintelligible rather than true or false.

2. This connection between Kuhn and the sociologists of scientific knowledge should not be taken to suggest that Kuhn himself was sympathetic to their project. In fact, he voiced serious reservations about the "historical philosophy of science" (Kuhn 1992). Further, as Friedman (1998) argues, the interpretation of Wittgenstein favored by sociologists of knowledge has little basis in the writings of Wittgenstein and has led to its embrace of an utterly implausible philosophical agenda.

The sociological reduction of mathematics and its history requires more than simply an appeal to forms of life as a way of characterizing mathematical communities. The sociologist of mathematical knowledge also wants to replace the usual account of mathematical objects and mathematical truth. A purely sociological approach to mathematics (especially that of the sort offered by Bloor) locates the objects of mathematical investigation in human social convention—they are literally "social constructs" brought into being by the system of human conventional behavior that underlies all meaning and implication. As Bloor puts it, the way to "give content" to the ordinary notion of mathematical objects and objectivity "is to equate it with the social" (Bloor 1991, 98). The sociological positivist's account of mathematical truth does not evaluate the truth of mathematical assertions "in terms of our practices 'corresponding' to some mysterious mathematical reality," but rather by reference to purely social factors (Barnes, Bloor, and Henry 1996, 185).

From the equation of mathematics and "the social" it follows fairly readily that there can be different mathematics, just as there are different patterns of social organization. It is no news to be told that languages, customs, political systems, religions, and other obviously social entities vary widely from culture to culture. Since he identifies the object of mathematics with institutionalized belief, Bloor's account requires that there be at least the possibility of such variation in mathematics. It is important to recognize that an alternative mathematics must consist of something deeper than just a different notation or a different way of developing familiar material. The alternative must literally contain different truths. For example, it is obvious and uninteresting that our system of arithmetical notation could have been different; in particular, we could have used the symbol '3' to designate the number four and '4' to designate the number three; then, assuming that the rest of our notation was unaltered, the expression '3 + 1 = 5' could have expressed a truth in arithmetic, namely the truth that four plus one is five. This kind of trivial semantic conventionalism is compatible with any view on the nature of mathematics. The sociological reductionist's commitment to the possibility of an alternative mathematics yields a much stronger consequence. It requires that the addition of the numbers three and one could yield the number five. Bloor readily admits this consequence, observing that an alternative mathematics would have to look like "error and inadequacy" from the standpoint of our mathematics:

> The "errors" in an alternative mathematics would have to be systematic, stubborn, and basic. Those features which we deem error would perhaps all be seen to cohere and meaningfully relate to one another by the practitioners of the alternative mathematics. They would agree with one another about how to respond to them; about how to develop them; about how to interpret them; and how to transmit their style of thinking to subsequent generations. The practitioners would have to proceed in what was, to them, a natural and compelling way. (Bloor 1991, 108)

If one grants the possibility of a genuinely alternative mathematics in the relevant sense, it follows quite easily that alternative systems should be studied impartially and "symmetrically," in the sense that no alternative should be accorded the privileged status of the incontestable truth about mathematics. Bloor's requirement of symmetry holds that a properly developed sociology of knowledge "would be symmetrical in its style of explanation. The same types of cause would explain, say, true and false beliefs" (Bloor 1991, 7).[3] Shapin and Schaffer employ considerations of symmetry when they consciously leave the category of "misunderstanding" out of their account of the controversy between Hobbes and Boyle: Hobbes's departures from Boyle's program in pneumatics were not a consequence of his *misunderstanding* experimental science; rather, he developed an alternative to it (Shapin and Schaffer 1985, 11–12). In the sociology of mathematics, the symmetry requirement bids the investigator not dismiss alternatives to "our" mathematics as incoherent, erroneous, or inconsistent. Applied specifically to the case of Hobbes's mathematical adventures, the sociologist of knowledge will have to describe Hobbes's deviation from the traditional view of the subject as the exploration of an alternative mathematics rather than a thoroughly confused lapse into incoherence and self-contradiction.

A sociological reductionist's account of Hobbes's mathematical career would, presumably, go something like this: Hobbes began as a member of the form of life known as the mathematicians' community, but he eventually embraced mathematical principles that were contrary to those of the group. These principles were a causal consequence of

3. Thus formulated, the symmetry requirement is utterly trivial. Inasmuch as they are all beliefs, both true and false beliefs have the same kind of cause, viz., whatever kind of cause it is that produces belief. I assume that Bloor intends a more finely grained causal typology than this, but will leave the issue aside here.

mitted error on quite a number of points. Indeed, as the debate between them raged on over the decades, it was obvious that they understood each other all too well. On any reasonable definition of what counts as a form of life, they were incontestably part of the same form of life: seventeenth-century intellectuals engaged in a "battle of the books" with conventions as explicit and well-understood as those of chess. Even in their disagreements about very general matters in the philosophy of mathematics (such as the nature of ratios or the relation between mathematics and natural philosophy), they understood what was being debated, the reasons advanced for each position, and the consequences attending to such reasons. Although one or the other disputant might dismiss his opponent's claims as "nonsense" or "unintelligible," even this does not show a significant failure of understanding. Wallis, for instance, understood perfectly well what Hobbes meant when he claimed that time and a line are homogeneous; but he took the claim to be false and dismissed it as "absurd," not thereby intending to indicate his inability to understand what was asserted, but rather to show that he took it to be simply false.

It is also critical to note that there is simply no interesting set of social factors that can put Hobbes's mathematical opponents into a single camp. By religion, class background, political affiliation, professional training, patronage, membership in scientific societies, or any other social category, those who criticized Hobbes's mathematics are a very diverse group indeed. These critics include Wallis, a Presbyterian divine and professor at Oxford; Huygens, a Dutch Calvinist of independent means who was elected to the Parisian Académie Royale des Sciences and granted a royal pension; André Tacquet, a Belgian Jesuit who taught at the universities of Bruges, Louvain, and Antwerp; Roberval, a French Catholic professor at the Collège Royal; de Sluse, a Belgian Catholic priest who held a variety of high offices in the church; Mylon, the son of Louis XIII's *contrôleur-général des finances* and an advocate at the *Parlement* of Paris; Pierre de Carcavi, a moderately wealthy French nobleman with no teaching position; John Pell, an English Protestant professor and clergyman who also held important government positions under the Protectorate of Cromwell; Viscount Brouncker, an Anglo-Irish aristocrat; and Seth Ward, an Anglican Royalist who nevertheless managed to hold the Savilian Professorship of Astronomy during the interregnum. It is only by reference to mathematical (as opposed to sociological) facts that one might hope to explain why such a diverse group could make common cause against

nature of the Trinity, the propriety of infant baptism, or the proper account of the Christian Sabbath provided him with the chance to engage in public controversy with a variety of authors that produced no fewer than eight published letters on the Trinity and numerous other minor pieces of polemical theology (Wallis 1692a, 1692b, 1696, 1697). Furthermore, Wallis became embroiled in a bitter dispute with William Holder concerning the credit for teaching a deaf-mute to pronounce some words of English. The result was his *Defence of the Royal Society* (Wallis 1678), which sharply rebuked Holder for the accusations in his *Supplement to the Philosophical Transactions of July 1670* (Holder 1678). Nor were Wallis's quarrels restricted to his published works; his letters clearly show him to have been a man eager for a fight. Scott reports that "Wallis was of a highly contentious disposition. His correspondence, unhappily, leaves no room whatever for doubt on that point. No man ever scorned personal popularity more completely than he" (1938, 88). It is far from credible that all of these quarrels could be correlated with some social or political interests, and the natural interpretation of Wallis's penchant for controversy must be in terms of individual psychological factors rather than some fanciful sociological just-so story in which he appears as the defender of a form of life. The relevance of this to the dispute with Hobbes should be obvious, as it shows that we need not seek for some underlying set of social interests to explain why Wallis pursued his battle with Hobbes so vigorously.

A final reason for treating the social factors as secondary in the dispute between Hobbes and Wallis is that they are not a constant. We saw in chapter 2 that the original impetus to quarrel involved the perceived threat that Hobbes posed to the universities. Yet the question of the status of English universities was settled well before the Restoration and disappeared from the exchanges between Wallis and Hobbes, while mathematical questions remained at center stage for decades. On the other hand, questions of political loyalty appeared relatively late in the dispute, and only after the mathematical terrain had been thoroughly worked over. Moreover, the key mathematical questions on which the dispute centered were not decided on the basis of social or political factors, but on straightforward mathematical grounds. Hobbes's numerous attempted quadratures and cube duplications won no adherents even among his friends and patrons, and the reason for this is evident: he was simply and spectacularly wrong. This is not to say that Hobbes failed in every aspect of his mathematics; after all (as we saw in chapter 4) he could pinpoint weaknesses in Wallis's mathematical work and his approach to questions in the philosophy of

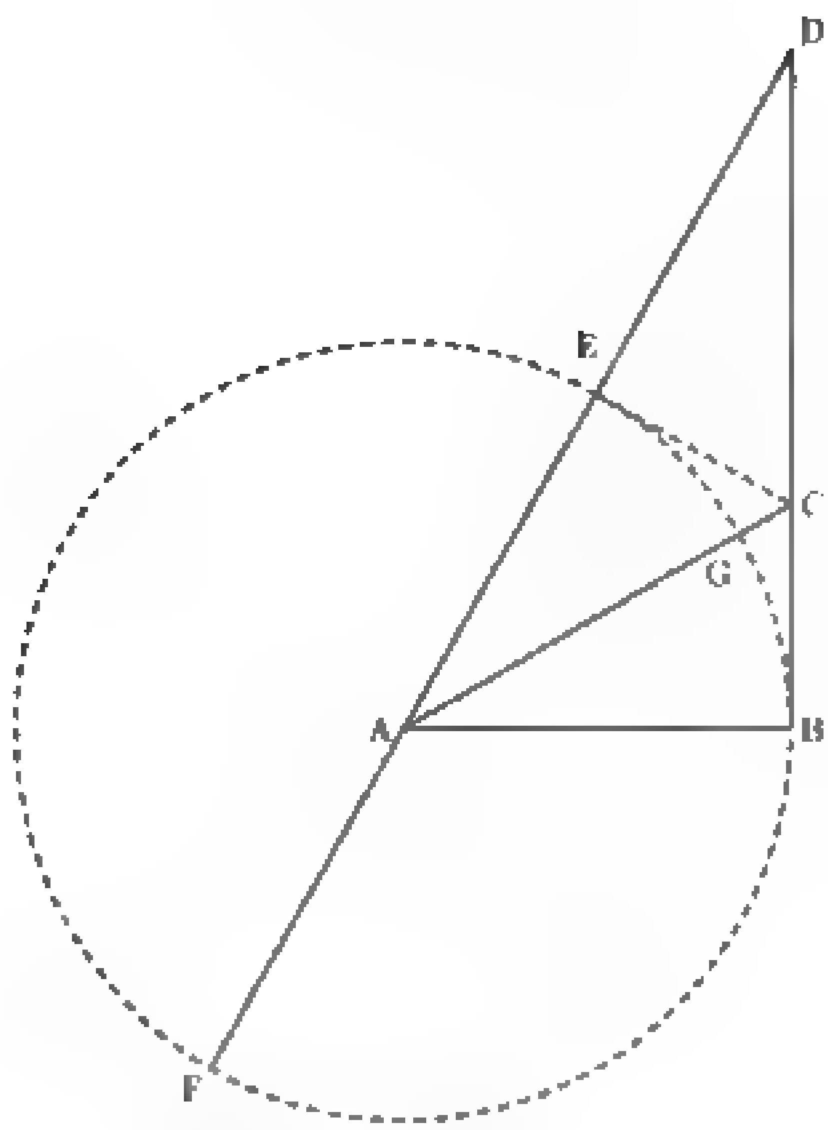

Figure A.1

latively short algebraic arguments, Hobbes's proof employs a rather prolix classical style characteristic of the "synthetic" demonstrations in Euclid's *Elements;* this is especially apparent in Hobbes's use of the various techniques for manipulating ratios and proportions, which he takes from the fifth book of Euclid. A manuscript version of Hobbes's proof survives as British Library MS. Add. 4278, f. 200r, from which it is possible to date the work to June of 1645. I have made minor changes to Hobbes's notation (these include using 'AB^2' where he uses '*ABq*', expressing proportions by '$A{:}B :: C{:}D$' rather than Hobbes's preferred symbolism '$A.B :: C.D$', using the symbol '×' for multiplication, and adding parentheses where necessary to identify the factors in a product).

The tangent to an arc less than a quadrant is to the tangent of half that arc as twice the square of the radius to the square of the radius minus the square of the tangent of one half the arc.

Let *EBF* [in figure A.1] be a circle with center *A* and radius *AB*, and let the arc less than a quadrant *AB* be *BE*, half of which is *BG*. The tangent of the whole arc *BE* is the right line *BD*. The tangent of the arc *BG* is the right line *BC*.

I say $BD{:}BC :: 2AB^2{:}AB^2 - BC^2$.

Let the points *E, C* be connected. Then because the right lines *AB, AC* of the triangle *ABC* are equal to the right lines *AE, AC* of the triangle *AEC*, and

$$\left\{\begin{array}{lcr} 2AB^2 & & \\ AB^2 & \ldots\ \ldots & DB \\ AB^2 - BC^2 & \ldots\ \ldots & 2BC \\ & & BC \end{array}\right\}$$

A.2 THE PRINCIPAL THEOREM IN THE QUADRATURE OF DEFICIENT FIGURES

The seventeenth chapter of *De Corpore* is devoted to the study of deficient figures produced by the motion of a line that diminishes as it moves through a given space. The determination of the area of such figures is the main goal of the chapter, and the key theorem asserts that if the ratio by which the line diminishes is in the *n*th power of the distance it has moved through the figure, then the ratio of the deficient figure to its complement is $1{:}n$. Expressed in more modern notation, this is a version of the result that $\int_0^a x^n dx = a^{n+1}/(n + 1)$. The remaining articles in chapter 17 of *De Corpore* contain a series of tables calculating the areas of various deficient figures and comparing them to one another. Wallis remarked that this chapter contains quite a few true propositions, notwithstanding the fact that the proof of the principal theorem is a failure. He concluded that, although he could not say where Hobbes had gotten them, "it is to be suspected that they are not yours, because they are true, while things of yours are wont to be false" (*Elenchus* 84).

Hobbes's presentation of this material has a very strong similarity to Cavalieri's method of indivisibles, most specifically in the fourth of his *Exercitationes Geometricae*, but it may also derive from Roberval's approach to the theory of indivisibles. One key point of similarity between Hobbes and the "indivisiblist" mathematicians is reflected in Hobbes's use of the language of indivisibles: he speaks of lines being drawn through "every possible part of a right line," refers to the latitude of small spaces as "indivisible," and describes a figure as "made up of so many indivisible spaces." The basic theorem is presented here in the version printed in the 1656 English translation of *De Corpore* and then in the version from the 1668 edition of *De Corpore*. The 1656 version contains two arguments for the result, the first of which was new to the English *De Corpore*, while the second is a reworked version of the

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consequent is to a third among the first magnitudes, so is a third to the antecedent among the second magnitudes." Hobbes's use of these concepts is relatively straightforward: the previous argumentation has established two sequences of ratios that can be compared to one another, and the middle terms of both sequences can be removed by using the Euclidean definitions.

argument in the original 1655 Latin *De Corpore*. The 1668 version gives a rather different version of the argument.

Although the various proofs differ in interesting ways, their similarities are also quite striking.[4] Hobbes attempts to establish his general result concerning the ratios of areas between a deficient figure and its complement, and in doing so he first attempts to establish that a ratio holds between pairs of lines in the deficient figure and its complement, after which he concludes that the same ratio will hold between the areas of the figure and its complement. This overall style of argument parallels the procedures of Cavalieri, but Hobbes's argumentation ultimately runs into serious difficulties in the details. The fact that the central result is given three distinct attempted proofs shows fairly clearly that Hobbes was made aware of the inadequacies in his argumentation, but (as inspection of the following material shows) the final version is hardly an improvement over those that preceded it. One important feature of Hobbes's argumentation is the fact that, although he speaks of figures as "made up" of indivisible elements, he does not attempt to represent the area of the figure as an infinite sum in the style of Roberval's *Traité des indivisibles* or Wallis's *Arithmetica Infinitorum*. Instead, he tries to work out a means of comparing the ratios of areas without using the sort of "algebraic" means he condemned as essentially ungeometrical.

A.2.1 De Corpore, *Part 3, Chapter 17, Article 2 (1656 Version)*[5]

A Deficient Figure, which is made by a Quantity continually decreasing to nothing by proportions every where proportionall and commensurable,[6] is to

4. Wallis took the differences between the Latin and English versions of chapter 17 of *De Corpore* as Hobbes's admission of error: "As to the Demonstration, you keep a vapouring (nothing to the purpose,) as if it were a good demonstration and not confuted. Yet, when you have done, (because you knew it to be naught) you leave it quite out in the English, and give us another (as bad) instead of it. That is, you confesse the charge. Your fundamentall Proposition was not demonstrated; and so this whole chapter comes to nothing" (*Due Correction* 119). This is a rather uncharitable reading of the evidence, since Hobbes did include the original argument (slightly modified in response to Wallis's criticisms) in the English version of *De Corpore*. Nevertheless, Wallis is correct in regarding Hobbes's argumentation as ultimately unconvincing.

5. I have corrected numerous typographical errors in the original and have made slight alterations to the diagram to make it fit the text. The frequency of typographical errors is particularly high in the second half of the article, which contains Hobbes's reworked version of the argument from the 1655 version *of De Corpore*. Almost all of these errors are in the labels for lines in the diagram. This suggests that in revising the argument to meet Wallis's criticisms Hobbes was either undecided about how best to proceed or making revisions hastily.

6. It should be remembered that Hobbes uses the English term *proportion* as a translation for the Latin *ratio*—a fact that leads to some significant linguistic confusions in

its Complement, as the proportion of the whole altitude, to an altitude diminished in any time, is to the proportion of the whole Quantity which describes the Figure, to the same Quantity diminished in the same time.

Let the quantity *AB* [in figure A.2.1], by its motion through the altitude *AC*, describe the Complete Figure *AD;* and againe, let the same quantity, by decreasing continually to nothing in *C*, describe the Deficient Figure *ABEFC,* whose Complement will be the Figure *BDCFE.* Now let *AB* be supposed to be moved till it lie in *GK*, so that the altitude diminished be *GC*, and *AB* diminished be *GE;* and let the proportion of the whole altitude *AC* to the diminished altitude *GC*, be (for example) triplicate to the proportion of the whole quantity *AB* or *GK*, to the diminished quantity *GE.*[7] And in like manner, let *HI* be taken equal to *GE*, & let it be diminished to *HF;* and let the proportion of *GC* to *HC* be triplicate to that of *HI* to *HF;* & let the same be done in as many parts of the straight line *AC* as is possible; and a line be drawn through the points *B, E, F* and *C*. I say the deficient figure *ABEFC*, is to its Complement *BDCFE* as 3 to 1, or as the proportion of *AC* to *GC* is to the proportion of *AB*, that is, of *GK* to *GE*.

For (by the second Article of the 15. Chap.) the proportion of the complement *BEFCD* to the deficient figure *ABEFC* is all the proportions of *DB* to *OE*, and of *DB* to *QF*, and of all the lines parallel to *DB* terminated in the line *BEFC*, to all the parallels to *AB* terminated in the same points of the line *BEFC*.[8] And seeing the proportions of *DB* to *OE*, and of *DB* to *QF* &c. are

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his account of the theory of ratios. Hobbes defines proportionality and commensurability of ratios at *DCo* 3.17.1; *EW* 1:247. The relevant definitions read as follows: "Four Proportions are said to be *Proportionall*, when the first of them is to the second, as the third is to the fourth. For example, if the first proportion be duplicate to the second; and again the third be duplicate to the fourth, those Proportions are said to be *Proportionall*. And Commensurable Proportions are those, which are to one another as number to number. As when to a proportion given, one proportion is duplicate, another triplicate, the duplicate proportion will be to the triplicate proportion as 2 to 3; but to the given proportion it will be as 2 to 1; and therefore I call those three proportions *Commensurable*." Despite some obscurity in the exposition, Hobbes's intent is fairly clear—when the ratio $\alpha:\beta$ is duplicate of $\gamma:\delta$, while the ratio $\varphi:\psi$ is likewise duplicate of $\chi:\omega$, then the pairs of ratios are proportional, which is essentially a generalization of the definition of four quantities standing in a proportion, i.e., in the same ratio. Similarly, commensurability is a matter of ratios being expressible in terms of integers.

7. The sixteenth article of chapter 13 of *De Corpore* (in the 1656 version) contains Hobbes's definition of multiplication of ratios: "A Proportion is said to be multiplied by a Number when it is so often taken as there be Unities in that Number" (*DeC* 2.13.16; *EW* 1:164). Thus, the ratio 1:3 taken twice is the ratio 1:9, or the ratio 5:4 taken twice is the ratio of 25:16; and in general if the ratio $\alpha:\beta$ is multiplied by n the result is $\alpha^n:\beta^n$. Thus, Hobbes is here assuming that $AC:GC = AB^3:GE^3$.

8. Hobbes here refers to a proposition added to the English version of chapter 15 that was not present in the 1655 Latin original. The proposition considers the velocities

Deficient Figures generated as above declared, the proportion of the Parallelogram to either of its parts; as that when the parallels encrease from a point in the same proportion, the Parallelogram will be divided into two equal Triangles; when one encrease is double to the other, it will be divided into a Semiparabola and its Complement, or into 2 and 1.

The same construction standing, the same conclusion may otherwise be demonstrated, thus.[11]

Let the straight line *CB* be drawn cutting *GK* in *L*, & through *L* let *MN* be drawn parallel to the straight line *AC*; wherefore the Parallelograms *GM* and *LD* will be equal. Then let *LK* be divided into three equal parts, so that it may be to one of those parts in the same proportion which the proportion of *AC* to *GC* or of *GK* to *GL* hath to the proportion of *GK* to *GE*. Therefore *LK* will be to one of those three parts as the Arithmetical proportion between *GK* and *GL* is to the Arithmetical proportion between *GK* and the same *GK* want the third part of *LK;* and *KE* will be somewhat greater then a third of *LK*.[12] Seeing now the altitude *AG* or *ML* is by reason of the continual decrease, to be supposed less than any quantity that can be given; *LK* (which is intercepted between the Diagonal *BC* and the side *BD*) will also be less then any quantity that can be given; and consequently, if *G* be put so neer to *A* in *g*, as that the difference between *Cg* and *CA* be less then any quantity that can be assigned, the difference also between *Cl* (removing *L* to *l*) and *CB*, will be less then any quantity that can be assigned; and the line *gl* being drawn & produced to the line *BD* in *k* cutting the crooked line in *e*, the proportion of *AC* to *gC* will still be triplicate to the proportion of *gk* to *ge*, and the difference between *k* and *e* and the third part of *kl* will be less then any quantity that can be given; and therefore the Parallelogram *eD* will differ from a third part of the Parallelogram *Ae* by a less difference then any quantity that can be assigned. Again, let *HI* be drawn parallel and equal to *GE*, cutting *CB* in *P*, the crooked line in *F*, and *EO* in *I*, and the proportion of *Cg*, to *CH* will be triplicate to the proportion of *ge* to *HF*, and *IF* will be greater then the third part of *PI*. But again,

11. What follows is a reworked version of the argument in the original 1655 *De Corpore*.

12. In the original version of this argument, Hobbes claimed that *KE* would be one-third of *LK* (Hobbes 1655, 145). Wallis objected that, since the point *G* is taken arbitrarily, the ratio between *KE* and *LK* could take on any desired value and the argument therefore fails (*Elenchus* 70). In the *Six Lessons*, Hobbes replied that "you did not then observe, that I make *the Altitude AG, less then any Quantity given*, and by consequence *EK* to differ from a third part by a less difference then any Quantity that can be given" (*SL* 5; *EW* 7:300). Hobbes therefore modified the argument by admitting that the ratio differs from the desired result, but that the difference can be made as small as desired.

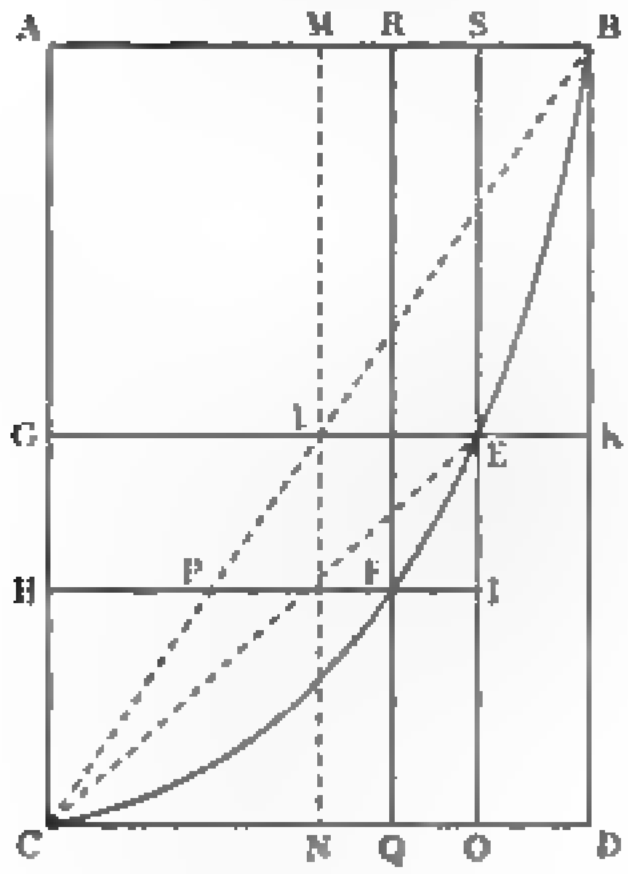

Figure A.2.2

then the space *ABEFC* is to the space *DCFEB* as one to one; but if the ratio of *AC* to *AG* is the duplicate of the ratio of *AB* to *GE*, then the space *ABEFC* is to the space *DBEFC* as two to one; if triplicate, as three to one, and so on.

LEMMA 1

By whatever ratio the velocity of a moved point is increased, in the same ratio are also increased the spaces passed over by the point in the same or equal times.

LEMMA 2

If between two right lines there are interposed an infinite number of both arithmetical and geometrical means, these do not differ in magnitude.[14]

In the parallelogram *ABCD* let the side *AB* be understood to be moved parallel to the side *CD*, and in moving to decrease until at last it evanesces in the point *C*; and by such motion the figure *ABEFC* is described, leaving the complement whose line *BEFC* is described by the endpoint *B* of the decreasing line *AB*. And in the same time let the side *AC* be understood to move uniformly to *BD*. Thus *CD* can be taken for the measure of time. But the right lines parallel to the line *CD*, terminated on one side in the line *BEFC*, and on the other in the right line *AC*, will be measures of the parts of time in which *AB* is moved to *CD* and *AC* to *BD*.

14. This is just the corollary to *DCo* 2.13.28 that Hobbes appealed to in his original argument, as well as in the revised version of that argument from 1656. The first lemma recapitulates principles from *DCo* 3.16, articles 1–6.

Now let there be taken arbitrarily in the right line *CD* a point *O*, and let *OS* be drawn parallel to the side *BD*, cutting the line *BEFC* in *E* and the right line *AB* in *S*. Again, from the point *Q* taken arbitrarily in *CD* let *QR* be drawn parallel to the same side *BD*, cutting *BEFC* in *F* and *AB* in *R*. And let *EG*, *FH* also be drawn, parallel to *CD* and cutting *AC* in *G* and *H*. Finally, let the same be supposed to be done in every point of the line *BEFC*.

I say that as the aggregate of all the velocities by which the right lines *QF*, *OE*, *DB*, and all the rest generated in the same manner is to the aggregate of the times designated by the right lines *HF*, *GE*, *AB*, and the rest, so the plane surface *DCFEB* is to the plane surface *ABEFC*. Just as *AB* by decreasing through the line *BEFC* in the time *CD* evanesces in point *C*, so *CD* (equal to *AB* itself) by decreasing through the same line *CFEB* in the same time evanesces in the point *B*, having described the right line *DB* equal to *AC*. Therefore the velocities with which *AC* and *DB* are described are equal to one another. On the other hand, seeing that in the same time in which the point *O* describes the right line *OE* the point *S* describes the right line *SE*, then *OE* will be to *SE* as the velocity with which *OE* is described to the velocity with which *SE* is described. And for the same reason *QF* will be to *RF* as the velocity with which *QF* is described to the velocity with which *RF* is described, and thus for all the other parallels. Therefore as the right lines that are parallel to the side *AB* and terminated in the line *BEFC* are the measures of the times, so the right lines that are parallel to the side *BD* (and terminated in the same line *BEFC*) are the measures of the velocities. Now (by lemma 1) in whatever ratio the velocities are increased, in the same ratio the right lines passed over in the same times are increased, namely *QF*, *OE*, *DB*, etc.

Now all the lines *QF*, *OE*, *DB*, etc. constitute the plane surface *DBEFC*; and all the lines *HF*, *GE*, *AB*, etc.—that is, all of *ES*, *FR*, *CA*, etc.—constitute the plane surface *ACFEB*. The former of these are the aggregate of the velocities, the latter the aggregate of times. Thus as the aggregate of the velocities is to the aggregate of the times, so the complement *DBEFC* is to the figure *ABEFC*. Therefore if indeed the ratios of *DB* to *OE* and *OE* to *QF* should be (for example) triplicate, then vice versa the ratios of *OE* to *DB*, and *QF* to *OE* will be subtriplicate of the ratios of *GE* to *AB* and *HF* to *GE*. Thus the aggregate of all of *QF*, *OE*, *BD*, etc. will be to the aggregate of all *HF*, *GE*, *AB*, etc. (by lemma 2) subtriple.[15] Therefore as the aggregate of the velocities is to the aggregate of the times by which the deficient figure is described, so will the complement of the figure to the deficient figure itself, that is to say the complement *DBEFC* to the figure *ABEFC*. Which was to be demonstrated.

15. Even the most charitable reading of Hobbes's principle leaves it obscure why this consequence should follow.

A.3 TWO OF HOBBES'S QUADRATURES FROM *DE CORPORE*, PART 3, CHAPTER 20

I here present two versions of the several attempted quadratures that Hobbes at various times intended for chapter 20 of *De Corpore*. The first is the original effort that Hobbes planned to include, but he abandoned when *De Corpore* was in proofs. Wallis acquired a copy of this from Hobbes's printer and quoted from it at great length (*Elenchus* 97–107). I have reconstructed the proof from Wallis's quotation of the unpublished sheets. The second is the first of three attempted quadratures published in the 1656 English version of *De Corpore*. Its construction is the same as that of the unpublished first quadrature, but Hobbes no longer claims it as an exact result. Instead, he declares that in his twentieth chapter "I have let stand there that which I did before condemn, not that I think it exact, but partly because the Division of Angles may be more exactly performed by it then by any organicall way whatsoever" (*SL* epistle; *EW* 7:186). Hobbes's indecision and frustration are palpable in the second half of the 1656 version of the argument: he grants that his construction conflicts with established values for π, but he cannot quite see the flaw in his procedures and is unwilling to let the matter rest. I find these two demonstrations more interesting than many other Hobbesian attempts at quadrature because they give a better picture of the "method of motions" he used to approach the problem and from which he expected such great results. This method attempts to rectify curvilinear arcs by imagining them to be straightened as points on the arc move uniformly through specified points in the construction. As is so often the case with Hobbes's mathematics, his constructions in both cases are quite complex (with a number of otiose lines cluttering the diagram) and the argumentation ultimately fails to achieve the desired result. Nevertheless, it is instructive to see just what sort of argument Hobbes had first found persuasive as he prepared chapter 20 of *De Corpore*.

A.3.1 The Original Quadrature Intended for De Corpore, *Part 3, Chapter 20, Article 1*

To find a right line equal to the perimeter of a circle.

Let *ABD* [in figure A.3.1] be the quadrant of a circle, about which is circumscribed the square *ABCD*. Let the sides of the square be bisected at *E, F, G,* and *H*. Then the lines *FH, EG* will divide $\widehat{BD}$ into three equal parts at *I* and *K*. Let *IM* (the sine of $\widehat{BI}$) be drawn, and *IM* will be half of the radius *BC*. Let $\widehat{BI}$ be divided into four equal parts at *L, N,* and *O*, and let the right line *MI* be so divided at *P, Q,* and *R*. Let *PL, QN,* and *RO* be connected, and to *SN* (the sine of $\widehat{BN}$) let *NT* be added, which is equal to *SN* itself. Finally, let *IT* be drawn and produced to meet *BC* in *V*. I say the right line *BV* is equal to

$\widehat{BI}$, and thus three times BV (which is Be) is equal to the arc BD, and twelve times the same BV is equal to the perimeter of the circle of which ABD is a quadrant.[16]

Because both MQ and QI, as well as SN and NT, are equal, then as ST is to SN, so MI is to QI. Let the right line TI be produced to meet BA produced in q. Then NQ produced falls on the same point q. And qP, qQ, qR joined and produced to BV will cut BV in four equal parts at X, Y, and Z.

On the right lines MQ, QI let two equilateral triangles MgQ, QbI be constructed, and with centers g and b let $\widehat{MQ}$, $\widehat{QI}$ be drawn. Either of these is equal to either of $\widehat{BN}$, $\widehat{NI}$.[17] Again, on the right lines MP, PQ, QR, RI let there be constructed as many equilateral triangles MkP, PlQ, QpR, RbI. And with centers k, l, p, b let $\widehat{MP}$, $\widehat{PQ}$, $\widehat{QR}$, $\widehat{RI}$ be drawn, any one of which is equal to any of $\widehat{BL}$, $\widehat{LN}$, $\widehat{NO}$, $\widehat{OI}$.

Now because $\widehat{IQ}$, $\widehat{IN}$ are equal, rectilinear motion through qQ places the point Q in N; and $\widehat{IN}$, $\widehat{IQ}$ will coincide, of course $\widehat{IQ}$ (which is slightly more curved than $\widehat{IN}$) being slightly straightened. And because $\widehat{IR}$, $\widehat{RQ}$ are both together equal to $\widehat{IN}$, these are straightened by the same motion and are placed in $\widehat{IN}$, with which they coincide. And thus by the rectilinear motion through qR the midpoint R, which is brought to the middle of the right line YV, will be brought to the middle of $\widehat{IN}$, that is through O.[18]

Similarly, because motion through qQ places the point Q in N, and motion through qM places the point M in B, and $\widehat{MQ}$, $\widehat{BN}$ are equal, $\widehat{MQ}$ coincides with $\widehat{BN}$. And by the same motion $\widehat{MP}$, $\widehat{PQ}$ will be placed in the same $\widehat{BN}$, with which they coincide. Therefore, the motion through qP places the midpoint P in the middle of $\widehat{BN}$, that is in L.[19]

In the same manner, by the perpetual bisection of the right line MI, and by constructing equilateral triangles on the parts thus produced, there will arise an infinity of arcs, that is, as many as one might wish, equal to each other

16. Elementary trigonemetric calculation shows that this assertion is false. By construction (taking the radius AB as a unit), $ST = 2\sin(15°)$, $MI = 1/2$, and $SM = \cos(15°) - \cos(30°)$. BV therefore has a length of approximately .5236539, and Hobbes requires a value for π at aproximately 3.1419234.

17. More precisely, taking AD as unit, $\widehat{BN} = \widehat{NI} = \pi/12 = \widehat{MQ} = \widehat{QI}$.

18. Wallis observes that this is the crucial misstep in the argument: "In no way [will the point R pass through O], for the right line qRZ does not pass through O. Although the points q, Q, N lie in the same right line, as also do q, M, B, nevertheless q, R, O do not, nor do q, P, L lie in the same line. And if you should contend that they do, it remains for you to prove it" (*Elenchus* 98). From here forward, the argument proceeds from a false supposition and delivers the inevitable false result.

19. Hobbes's "method of motions" fails here again. He has no guarantee that qP continued will pass through L, and calculation shows that, indeed, it does not. In effect, he is assuming what he needs to prove, namely that the fourth part of the line BV is equal to a fourth part of $\widehat{BI}$.

Let *aL* (the sine of $\widehat{BL}$) be drawn, and let it be produced until it cuts *IV* at *c*. Then, because *MP* is the fourth part of *MI*, *aL* will be the fourth part of *ac*. And because *qB* is greater than *qa*, *BV* will also be greater than *ac*. Therefore *BV* is greater than four sines of $\widehat{BL}$, which arc is one fourth of $\widehat{BI}$. In the same way it can be shown that if $\widehat{BI}$ were divided into any number of equal parts (so that the difference between the arc itself and the aggregate of as many sines of one of these smallest parts as there are parts in the division is less than any given quantity), the right line *BV* would still be greater than all of these sines taken together. Therefore the right line *BV* is not less than the arc *BI*. But it cannot be greater either, because if this point *B* itself is taken for the sine of the smallest part of the arc, then the aggregate of all the sines considered as points is the right line *BV* itself, and it is equal to the arc *BI*.[21] Therefore the right line *BV* is equal to the arc *BI*, and *Be* is equal to the arc *BD*, and four times *Be* is equal to the perimeter of the circle of which *ABD* is a quadrant. Therefore a right line has been found equal to the perimeter of the circle, which was to be done.

A.3.2 The First Quadrature from the 1656 De Corpore

Let the Square *ABCD* [in figure A.3.2] be described, and with the *Radii AB*, *BC* and *DC* the three Arches *BD*, *CA* and *AC*; of which let the two *BD* and *CA* cut one another in *E*, and the two *BD* and *AC* in *F*. The Diagonals therefore *BD* and *AC* being drawn will cut one another in the center of the Square *G*, and the two Arches *BD* and *CA* into two equal parts in *H* and *Y*; and the Arch *BHD* will be trisected in *F* and *E*. Through the Center *G* let the two Straight Lines *KGL* and *MGN* be drawn parallel and equal to the sides of the Square *AB* and *AD*, cutting the four sides of the same Square in the points *K*, *L*, *M* and *N*; which being done, *KL* will pass through *F*, and *MN* through *E*. Then let *OP* be drawn parallel and equal to the side *BC*, cutting the Arch *BFD* in *F*, and the sides *AB* and *DC* in *O* and *P*. Therefore *OF* will be the sine of the arch *BF*, which is an arch of 30 degrees; and the same *OF* will be equal to half the Radius. Lastly, dividing the arch *BF* in the middle in *Q*, let *RQ* the Sine of the

for the quadrature of the circle. Hobbes continues, however, and the remaining argumentation resembles the kind of double reductio ad absurdum argument found in Archimedean exhaustion proofs. His idea is to show that the line *BV* can be neither greater nor less than the arc *BI*. Unfortunately for Hobbes, his argument begs the crucial question.

21. This part of the argument begs the question, as Hobbes later realized. In the printed version of the 1655 *De Corpore* he admitted, "Seeing that it is possible that the line *qP* produced to the perpendicular *Li* may cut it beyond *L*, it is also possible that *aL* is greater than one-fourth of the right line *ac*. Therefore it is not demonstrated that the right line *BV* is equal to the arc *BI*" (Hobbes 1655, 171).

having their cavity towards the same parts; which how it should be, unlesse all those small arches should be coincident with the arch *BF* in all its points, is not imaginable.[25] They are therefore coincident, and all the straight lines drawne from *X* & passing through the points of division of the straight line *OF*, will also divide the arch *BF* into the same proportions into which *OF* is divided.[26]

Now seeing *Xb* cuts off from the point *B* the fourth part of the arch *BF*, let that fourth part be *Be*; and let the Sine thereof *fe* be produced to *FT* in *g*, for so *fe* will be the fourth part of the straight line *fg*, because as *Ob* is to *OF*, so is *fe* to *fg*. But *BT* is greater then *fg*; and therefore the same *BT* is greater then four Sines of the fourth part of the arch *BF*. And in like manner, if the arch *BF* be subdivided into any number of equal parts whatsoever, it may be proved that the straight line *BT* is greater then the Sine of one of those small arches so many times taken as there be parts made of the whole arch *BF*. Wherefore the Straight line *BT* is not lesse then the Arch *BF*. But neither can it be greater, because if any straight line whatsoever, lesse then *BT*, be drawn below *BT* parallel to it and terminated in the straight lines *XB* and *XT*, it would cut the arch *BF*; and so the Sine of some one of the parts of the arch *BF* taken so often as that small arch is found in the whole arch *BF*, would be greater then so many of the same arches; which is absurd.[27] Wherefore the Straight line *BT* is equal to the arch *BF*, & the Straight line *BV* equal to the Arch of the Quadrant *BFD*; and *BV* four times taken, equal to the Perimeter of the Circle described with the Radius *AB*. Also the Arch *BF* and the Straight line *BT* are every where divided into the same proportions; and consequently any given Angle, whether greater or less then *BAF* may be divided into any proportion given.

But the straight line *BV* (though its magnitude fall within the terms assigned by *Archimedes*) is found, if computed by the Canon of Sines, to be somwhat

scribes here will undoubtedly straighten the small arcs, he has no guarantee that they will be brought into coincidence with $\widehat{BF}$, and this fact is independent of his rather jarring assumption that the line *OF* can be infinitely subdivided and an infinity of small arcs produced.

25. Hobbes evidently felt some unease about the soundness of this argumentation, but he could not give up the idea that he had found the way to rectify $\widehat{BF}$. His assertion that the two "crooked lines" are equal to $\widehat{BQ}$ and $\widehat{QF}$ is, however, in error and begs the very question at issue.

26. As with the previous attempt at quadrature, the demonstration could have ended here, since this result would be equivalent to the quadrature of the circle. Hobbes nevertheless continues with the same kind of flawed attempt at double reductio ad absurdum he had earlier undertaken.

27. The "absurdity" arises from the fact that Hobbes in effect assumes that his construction divides the arc into equal parts, which is exactly the result he needed to prove. In fact, *BT* will exceed the arc $\widehat{BF}$.

ble to *BT*, the line *Xb* shall not cut off the arch *BE* which is double to the arch *BF*, but a much greater. For the magnitude of the straight lines *XM*, *XB* and *ME* being known (in numbers) the magnitude of the straight line cut off in the Tangent by the straight line *XE* produced to the Tangent may also be known; and it will be found to be less then *Bb;* wherefore the straight line *Xb* being drawn will cut off a part of the arch of the Quadrant greater then the arch *BE*. But I shall speak more fully in the next Article concerning the magnitude of the arch *BI*.

And let this be the first attempt for the finding out of the dimension of a Circle by the Section of the arch *BF*.

A.4 THE COMPARISON OF THE SPIRAL OF ARCHIMEDES WITH THE PARABOLA

This result is one of the more intriguing pieces of mathematics to make its way into *De Corpore*. It is Hobbes's account of the rectification of the Archimedean spiral in part 3, chapter 20, article 5. As I argued in chapter 3, the reasoning Hobbes employs here is closely connected with Roberval's analysis of the same problem, as well as making use of Galileo's analysis of the construction of the parabola from uniformly accelerated motion. The argument is essentially sound, the only flaw being Hobbes's assumption that he has rectified arc of the quadrant (in the preceding sections) and that he had found the means to rectify the parabola in the eighteenth chapter of *De Corpore*. This led Hobbes to declare that

> From the known Length of the Arch of a Quadrant, and from the proportional Division of the Arch and of the Tangent *BC*, may be deduced the Section of an Angle into any given proportion; as also the Squaring of the Circle, the Squaring of a given Sector, and many the like propositions, which it is not necessary here to demonstrate. I will therefore onely exhibit a Straight line equal to the Spiral of Archimedes, and so dismiss this speculation. (*DCo* 3.20.4; *EW* 1:307)

The argument remained unaltered between its first appearance in the 1655 and 1656 versions of *De Corpore*, but it was removed from the 1668 version. I present the English translation of 1656.

The length of the Perimeter of a Circle being found, that Straight line is also found, which touches a Spiral at the end of its first conversion. For upon the center *A* [in figure A.4] let the circle *BCDE* be described; and in it let *Archimedes* his Spiral *AFGHB* be drawn, beginning at *A* and ending at *B*. Through the center *A* let the straight line *CE* be drawn, cutting the Diameter *BD* at

crooked line of a Parabola will be described.[33] Again, if the point *A* be conceived to be moved uniformly in the straight line *AB*, and in the same time to be carried round upon the center *A* by the circular motion of all the points between *A* and *B*, *Archimedes* has demonstrated that by such motion will be described a Spiral line. And seeing the circles of all these motions are concentrick in *A;* and the interiour circle is alwayes lesse then the exteriour in the proportion of the times in which *AB* is passed over with uniform motion; the velocity also of the circular motion of the point *A*, will continually encrease proportionally to the times. And thus far the generations of the Parabolical line *MK*, and of the Spiral line *AFGHB*, are like. But the Uniform motion in *AB* concurring with circular motion in the Perimeters of all the concentrick circles, describes that circle, whose center is *A*, and Perimeter *BCDE*; and therefore that circle is (by the *Coroll.* of the first article of the 16 Chapter) the aggregate of all the Velocities together taken of the point *A* whilst it describes the Spiral *AFGHB*.[34] Also the rectangle *IKLM* is the aggregate of all the Velocities together taken of the point *M*, whilest it describes the crooked line *MK*. And therefore the whole velocity, by which the Parabolicall line *MK* is described, is to the whole velocity with which the Spiral line *AFGHB* is described in the same time, as the rectangle *IKLM*, is to the Circle *BCDE*, that is to the triangle *AIB*. But because *AI* is bisected in *K* & the straight lines *IM* & *AB* are equal, therefore the rectangle *IKLM* and the triangle *AIB* are also equal. Wherefore the Spiral line *AFGHB*, and the Parabolical line *MK*, being described with equal velocity and in equal times, are equal to one another. Now

33. This is the familiar result from the "fourth day" of Galileo's "Two New Sciences" (Galileo 1974, 221).

34. The corollary in question is Hobbes's version of the "mean speed theorem" and asserts that "[i]f the *Impetus* be the same in every point, any straight line representing it may be taken for the measure of Time; and the Quicknesses or *Impetus* applied ordinately to any straight line making and Angle with it, and representing the way of the Bodies motion, will designe a parallelogram which shall represent the velocity of the whole Motion. But if the *Impetus* or Quickness of Motion begin from Rest, and increase Uniformly, that is, in the same proportion continually with the times which are passed, the whole Velocity of the Motion shall be represented by a Triangle, one side whereof is the whole time, and the other the greatest *Impetus* acquired in that time; or else by a parallelogram, one of whose sides is the whole time of Motion, and the other, half the greatest *Impetus;* or lastly by a parallelogram having for one side a mean proportional between the whole time & half of that time, & for the other side the half of the greatest *Impetus*. For both these parallelograms are equal to one another, & severally equal to the triangle which is made of the whole line of time, and the greatest acquired *Impetus;* as is demonstrated in the Elements of Geometry" (*DCo* 3.16.1, corollary 2; *EW* 1:219). Hobbes's use of it here is to argue that the increasing velocity of the point describing the spiral can be analyzed as a right triangle, having one side equal to the radius of the circle and the other equal to the circumference. Such a triangle has an area equal to that of the circle.

cavity of the circle *VXYZ* will fall on *Z*, because *ST*, *TD* are equal. Consequently, *SZ* will be equal to *DX*, and *XZ* will be a diameter of the circle *VXYZ*. Therefore, the angle *XYZ* in the semicircle will be a right angle, and in drawing the right lines *YZ*, *VZ* we make the rectangle *VXYZ*, whose sides *VX*, *YZ* are parallel.[37]

Now, if the right line *YZ* produced falls on *P*, the whole line *PZY* will be right and parallel to *VX*, and the alternate angles *YPX*, *VXP* will be equal. The angles *YPX* and *XYD* will also be equal, and the three right triangles *PDY*, *YDX*, and *XDV* will be similar. Consequently, the four right lines *PD*, *DY*, *DX*, *DV* will be in the same continued ratio.

It is therefore required to demonstrate that the right line *YZ* produced falls on *P*.

Let *PV* be drawn and divided in two equal parts at *a*. Let the right line *ab* also be drawn parallel to *AV* and cutting *PD* at *c*. Further, let *Td* be drawn parallel to *PD*, cutting *ab* at *d*, and let *dc* be divided in two equal parts at *g*. On the center *g* with distance *ga* let the semicircle *ahb* be described, cutting *PD* at *h* and *ab* at *b*.

This being done, the two right lines *ah*, *bh* being drawn will form a right angle at *h*. Now *ac* is half of *DV*, and because *dg*, *gc* are equal, *db* will also be equal to half of *DV*, and *ab* will be half of *YV*.

Therefore, as *PD* is to *DY*, that is to say to the line composed of *DS* and *SY*, so also is *Pc* (the half of *PD*) to *cb*, the line composed of the halves of *DS* and *SY*, and consequently *Pb* being produced will fall on *Y*. And the right lines *hb*, *ha* will be the halves of the right lines *XY*, *XV*; and *XY* being divided in two equal parts at *i*, the figure *Yihb* will be a rectangle, and *Yb* will be parallel to *XV*.[38] But *YZ* is parallel to *XV*. Therefore, *YZ* produced will fall on *P*. And (because of what was demonstrated) the four right lines *PD*, *DY*, *DX*, *DV* are in one and the same continued ratio. I have therefore found two mean proportionals between a given right line and its half. Which was required to be done.

Consectary. A cube that has the lesser of these two means as an edge is double of a cube that has the half of the greater extreme for an edge.[39] Because the ratio of a cube to a cube is triplicate of the ratio of the edge to an edge. And the ratio of *PD* to *DX* is triplicate of the ratio of *PD* to *DY*.

37. Reading "tirant les doites YZ, VZ" for "tirant la doite YZ."

38. This consequence fails to follow. The prior argumentation establishes that *Yihb* is a parallelogram, but not that it is a rectangle. From this point forward, the demonstration proceeds from a false assumption to a false result.

39. Reading, "Vn Cube qui a pour côté la moindre de ces deux moyennes" for "Vn Cube qui a pour côté la plus grande de ces deux moyennes," which misstates the intended result.

triangles *ACG*, *AYQ* are parallel, the base *YQ* is also bisected at *P*, and thus the triangles *AYP*, *APQ* are equal.

In $\widehat{LC}$ let $\widehat{LV}$ be taken equal to $\widehat{CP}$, and let *AV* be drawn, cutting *YP* at *X*.

Now $APL + PQL + CYP = AVL = ACP$ (because $APL + PQL = AYP$). Also, $ACV + AVP = ACP = AVL$.

Thus $APL + PQL + CYP = ACV + AVP$.

Subtracting the equals *APL*, *ACV* from both sides, there remains $PQL + CYP = AVP$.

Therefore because the sector *AVP* added to the equal sectors *ACV*, *ALP* makes the whole sector *ACL*, so also the two trilinear figures *PQL*, *CYP* added to the same equal sectors *ACV*, *ALP* make two equal triangles equal to the same sector *ACL*.[40] Now the trilinear figure *PQL* added to the sector *ALP* makes the triangle *APQ*. And (because the sectors *ALP*, *ACV* are equal and triangles *AYP*, *APQ* are equal) the same trilinear figure *PQL* added to the sector *ACV* makes the triangle *AYP*.

Therefore if *PQL*, *CYP* are equal, the whole triangle *AYQ* will be equal to the whole sector *ACL*. But if *PQL* is greater or less than *CYP*, the triangle *AYQ* will be greater or less than the sector *ACL*. Therefore either no right triangle can be taken with vertex *A* and equal to the sector *ACL*, or *PQL* and *CYP* are equal.[41] But the first is absurd. Therefore, *PQL*, *CYP* are equal, of which the first *(PQL)* extends wholly outside the sector *ACL*, while the second *(CYP)* is wholly contained within the sector *ACL*.

Thus the triangles *AYP*, *APQ* taken together (that is an eighth part of the whole square *QRST*) are equal to the two sectors *ACP*, *APL* taken together (that is to an eighth part of the whole given circle *BCDE*), and the whole square *QRST* is equal to the whole circle *BCDE*.

Therefore a square has been found equal to a given circle.[42]

40. As Wallis observes (1669a, 2), this misstates the case slightly since "these will not *make two triangles* (even though they can be equal to two triangles)."

41. Sadly for Hobbes, both disjuncts of this claim are false.

42. Elementary calculation reduces this conclusion to absurdity. Since $AO:AP :: AC:AY :: OC:PY$, it follows that $AO^2:AP^2 :: ACO:AYP$ (since similar figures with proportional sides are in duplicate ratio). Then, setting the radius *AP* or *AC* equal to *R*, we get $CO = R/2$, and $AP^2 = R^2$. Further, $AO^2 = R^2 + R^2/4 = 5R^2/4$ and (substituting and simplifying the proportion $AO^2:AP^2 :: ACO:AYP$) we obtain $ACO = R^2/4$. Thus, $AYP = R^2/5$ and $AYQ = 2R^2/5$. In consequence *QRST* will be $16R^2/5$, which results in a value of 3.2 for π.

PHILOSOPHY / SCIENCE

In 1655, the philosopher Thomas Hobbes claimed he had solved the centuries-old problem of "squaring the circle"—constructing a square equal in area to a given circle. With a scathing rebuttal to Hobbes's claims, the mathematician John Wallis began one of the longest and most intense intellectual disputes of all time. *Squaring the Circle* is a detailed account of this controversy, from the core mathematics to the broader philosophical, political, and religious issues at stake.

Hobbes believed that by recasting geometry in a materialist mold, he could solve any problem in geometry and thereby demonstrate the power of his materialist metaphysics. Wallis, a prominent Presbyterian divine as well as an eminent mathematician, refuted Hobbes's geometry as a means of discrediting his philosophy, which Wallis saw as a dangerous mix of atheism and pernicious political theory.

Hobbes and Wallis's "battle of the books" illuminates the intimate relationship between science and crucial seventeenth-century debates over the limits of sovereign power and the existence of God.

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